EFFECT OF RBF BASIS FUNCTIONS ON STRUCTURAL RELIABILITY ESTIMATES

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ABSTRACT

Traditional approaches such as the Monte Carlo simulation and the finite element method are widely used for structural reliability analysis but usually demand excessive computational resources. To address this limitation, response surface methods employing radial basis functions (RBFs) have been introduced as efficient surrogates for approximating implicit limit state functions. Nevertheless, the accuracy of RBF-based reliability analysis strongly depends on the choice of parameters and basis functions, while systematic guidelines for their selection remain insufficient. This study investigates how different RBF types—Gaussian (GA), Multi-Quadric (MQ), Inverse Multi-Quadric (IMQ), Thin Plate Spline (TPS), Cubic, and Linear—and their parameter settings influence reliability index assessment (RIA). The surrogate models are constructed using Latin Hypercube Sampling (LHS) to approximate limit state functions, and the Hasofer–Lind–Rackwitz–Fiessler (HL-RF) algorithm is employed to compute both the reliability index and failure probability. The findings provide insights into the sensitivity of structural reliability estimates to RBF configuration, offering useful guidance for selecting appropriate surrogate modeling strategies in engineering applications.

Keywords: structural reliability analysis; first-order reliability method; response surface method; radial basis function

1. INTRODUCTION

In structural reliability analysis method based on radial basis function and PMA-based reliability analysis technique using radial basis function are proposed. Radial basis functions have become an important weapon in computer graphics and adaptive numerical solutions to differential equations. However, the parameters of radial basis functions effect to the reliability analysis results, but, there is no guidance how to select appropriate values for the parameters.

Historically, polynomial functions have been the most common choice for constructing response surfaces. Bucher and Bourgund (1990) [1] proposed an adaptive interpolation scheme using second-order polynomials without mixed terms, which represent the original function primarily along the coordinate axes. This approach requires a relatively low number of numerical experiments (2n+1 parameters) to obtain a unique response surface. However, this traditional polynomial-based RSM can encounter limitations. Its accuracy is highly sensitive to the selection and placement of sampling points; for instance, the arbitrary factor k_i used to select sampling points can significantly influence the failure probability estimate. Small k_i values might lead to overestimation, while larger values could result in underestimation of failure probability, and very small k_i can even cause numerical instability. Furthermore, simple polynomials may struggle to accurately capture highly nonlinear limit states, and increasing the polynomial order to improve the fit can introduce non-physical oscillations, require an excessive number of support points, and lead to ill-conditioned systems of equations. The inclusion of mixed terms, though sometimes crucial for accuracy, also significantly increases the number of required experiments.

To overcome these limitations, researchers have explored various advancements and alternative basis functions for response surface construction. Adaptive and sequential RSM approaches iteratively refine the response surface by relocating sampling points closer to the limit state surface. Kim and Na (1997) [2] introduced a gradient projection technique for sampling point selection, adapting the sampling range based on the detected nonlinearity of the limit state. They demonstrated that their proposed method, utilizing linear response surface functions, yielded accurate results for both linear and mildly nonlinear limit states. Zheng and Das (2000) [3] suggested a cumulative formation approach, progressively adding square and cross terms to the response surface. Their work indicated that while linear response surfaces could be less accurate, a response surface with square terms (g(x)) closely approximated the true limit state function, and one with both square and mixed terms ($\hat{g}(x)$) could be almost identical to it. Gayton et al. (2003)[4] developed CQ2RS, a statistical RSM utilizing resampling and confidence intervals to validate results and guide the selection of new experiments, aiming to reduce computational cost for complex mechanical problems. More recently, Gavin and Yau (2008)[5] introduced the High-Order Stochastic Response Surface Method (HO-SRSM), which approximates the limit state function using arbitrary order polynomials. By employing Chebyshev polynomials for sampling and determining polynomial orders based on statistical significance of terms, HO-SRSM showed more consistent probability of failure (Pf) estimates across varying sampling ranges (h) compared to traditional second-order SRSM, which is highly dependent on h.

Various surrogate modeling techniques have been extensively developed and applied to address implicit and highly nonlinear performance functions in structural reliability analysis. Among them, radial basis function (RBF) networks have attracted considerable attention due to their strong approximation capability and computational efficiency. RBF-based approaches, including RBF-MCS, RBF-FORM, and RBF-SORM, have demonstrated accuracy comparable to conventional reliability methods while enabling efficient evaluation of function values and their derivatives. Deng (2006) [6] reported that approximation errors are smallest for the original performance function, increase for first derivatives, and are largest for second derivatives. Commonly used RBFs include multiquadric, inverse multiquadric, and Gaussian functions. To further enhance performance, Mullur and Messac (2005) [7] proposed the extended RBF (E-RBF), which outperforms traditional RBF and Kriging models in terms of accuracy, robustness to sampling strategies, and effectiveness in high-dimensional spaces, without requiring extensive parameter tuning. The use of three independent coefficients per dimension provides E-RBF with additional flexibility.

Artificial neural networks (ANNs) have also been widely employed as universal function approximators capable of capturing complex nonlinear relationships among input variables. Cheng et al. (2007) [8] successfully applied ANN models to approximate structural response functions, significantly reducing the number of deterministic finite element analyses and enabling explicit formulations for inverse FORM. Similarly, Bucher and Most (2008) [9] highlighted the adaptability and flexibility of ANNs in modeling input–output relationships in reliability analysis.

Kriging-based surrogate models combine a global regression component with a localized Gaussian stationary process, ensuring exact interpolation at sampled points. Kaymaz (2005) [10] demonstrated that Kriging generally outperforms classical response surface methods, particularly when a sufficient number of experimental points is available. Nevertheless, its performance is sensitive to the selection of correlation parameters and may deteriorate in high-dimensional problems, necessitating careful parameter optimization. Moving least squares (MLS) approximation represents another advancement in surrogate modeling. Kang et al. (2010) [11] proposed an MLS-based response surface method that improves upon traditional least squares by assigning higher weights to experimental points closer to the Most Probable Failure Point (MPFP). This localized weighting strategy enhances the accuracy of the response surface near the MPFP, leading to faster convergence and improved reliability estimates with fewer function evaluations.

More recently, ensemble metamodeling techniques have been proposed to further enhance predictive accuracy and robustness. Acar and Solanki (2009) [12] showed that combining multiple surrogate models such as polynomial response surfaces, RBFs, Kriging, Gaussian processes, and support vector regression into an ensemble framework can outperform even the best individual metamodel. Their study also emphasized that the optimal surrogate model is problem-dependent, highlighting the advantage of leveraging multiple models in structural reliability analysis.

Moran et al.(2023)[25] presented by at the ICASP14 conference, addresses the challenge of efficiently estimating multiple performance functions in engineering applications. The authors utilize PC-Kriging-based surrogate models and introduce a variance correction derived from leave-one-out cross-validation to enhance the accuracy of the sequential learning process, which is guided by U-function-derived metrics. The effectiveness of these proposed strategies is demonstrated through a highly nonlinear analytical reliability setting and a practical application involving ship collisions with offshore wind substructures. The results indicate that by targeting multiple limit states interchangeably, the proposed methods achieve more accurate and balanced predictions for various structural events compared to single-target approaches.

The diverse range of approximate functions and metamodeling techniques highlights a critical aspect of RSM: the choice of the underlying basis function significantly impacts the quality of the response surface and, consequently, the accuracy, stability, and efficiency of the structural reliability estimates. While advanced methods promise better performance, their efficacy can be problem-dependent, influenced by factors such as the degree of nonlinearity, the number of random variables, and their correlation properties.

Based on the above review, it can be concluded that the response surface method constitutes a powerful and widely used framework for structural reliability analysis; however, the selection of an appropriate basis function for approximating the limit state function remains a critical and nontrivial issue. Traditional polynomial-based response surfaces, although computationally simple and extensively adopted, often suffer from limited accuracy when applied to highly nonlinear problems and may exhibit numerical instability depending on the sampling strategy and the choice of control parameters. In contrast, more advanced approximation techniques such as radial basis functions (RBFs), artificial neural networks (ANNs), Kriging, and moving least squares (MLS)—have demonstrated notable improvements in accuracy, robustness, and computational efficiency by more effectively capturing complex nonlinear behavior and concentrating approximation efforts in critical regions of the design space. More recently, ensemble metamodeling approaches have further highlighted that no single surrogate model is universally optimal, as different techniques exhibit complementary strengths depending on the underlying problem characteristics. Nevertheless, the performance of these advanced basis functions remains highly problem-dependent, and systematic comparative assessments across different structural reliability scenarios are still limited.

Therefore, in this study, an enhanced reliability analysis framework is developed to evaluate and compare the performance of various radial basis function (RBF) models, including Gaussian, Multi-Quadric, Inverse Multi-Quadric, Thin Plate Spline, Cubic, and Linear functions. The influence of the shape parameter α_i in the response surface formulation is systematically examined by assigning multiple values and observing its impact on the computed reliability index and failure probability. The proposed approach integrates RBF-based surrogate modeling with the Reliability Index Approach (RIA), where the limit state functions are approximated using Latin Hypercube Sampling (LHS). The most probable point (MPP) is subsequently determined by applying RIA to the constructed response surfaces.

2. MATERIALS AND RESEARCH METHODS

2.1 Construction of radial basis functions

Radial basis functions (RBFs) were originally introduced as an efficient tool for interpolating scattered multivariate data. In this approach, the response function is approximated through a weighted sum of radially symmetric kernels defined with respect to the Euclidean distance or alternative distance measures. Several types of RBFs have been proposed in the literature, each offering distinct characteristics and approximation capabilities.

The following choices of RBF are considered [13] [23] [24]:

Multi-Quadric (MQ): $\Phi(r) = \sqrt{r^2 + c^2}$

Inverse Multi-Quadric (IMQ) : $\Phi(r) = 1/\sqrt{r^2 + c^2}$

Thin Plate Spline (TPS): $\Phi(r) = r^2 \ln r$

Cubic: $\Phi(r) = r^3$

Linear: $\Phi(r) = r$

The radial basis function interpolation problem is as follows. Let n pairwise different points $X_1, X_2, \ldots, X_n \in R^d$ and data $f_1, f_2, \ldots, f_n \in R$ be given, where *n* and d are any positive integers. A function of the following simple form [13]

$$\tilde{g}(X) = \sum_{i=1}^{n} \lambda_i \phi(\|X - X_i\|), \quad X \in R_d \tag{1}$$

that interpolates the data $(X_1, f_1), (X_2, f_2), \ldots, (X_n, f_n)$. The coefficients λ_i, i=1,2,…,n are real numbers, and the norm ‖.‖ is the Euclidean in R^d. The matrix $\Phi\,\Phi \in R^{n\times n}$ that is defined by

$$[\Phi]\{\lambda\} = \{G\} \tag{2}$$

where

$$\Phi_{ij} = \phi(\|X_i - X_j\|), \quad \mathrm{i, j} = 1,2,\ldots,\mathrm{n} \tag{3}$$

$$\{\lambda\} = [\lambda_1 \ \lambda_2 \cdots \lambda_n]^T \tag{4}$$

$$\{G\} = [g(X_1)\ \mathrm{g}(\mathrm{X}_2)\cdots \mathrm{g}(\mathrm{X}_n)]^T \tag{5}$$

2.2 Reliability estimates in reliability index approach (RIA)

Consider following limit state function with *n* uncertain parameters

$$g(\mathbf{X}) = 0, \quad X_i = 1, 2, \cdots, n \tag{6}$$

The statistical description of the failure of the limit state function $g(\mathbf{X})$ is characterized by the Cumulative Distribution Function (CDF) $F_g(0)$ as

$$P(g(\mathbf{X}) \le 0) = F_g(0) \tag{7}$$

The CDF is defined as

$$F_g(0) = \int_{g(\mathbf{X})\le 0} f_{\mathbf{X}}(\mathbf{X}) d\mathbf{x} \tag{8}$$

Where $f_x(\mathbf{X})$ is the joint probability density function

The first-order reliability index β (Fig 1) is computed using the FORM that is formulated as a reliability analysis problem with one equality constraint in **U**-space, which is defined as the limit state function [11, 14, 15]:

$$\begin{aligned} &\min && \|\mathbf{U}\| \\ &\text{Subject to} && \mathrm{g}(\mathbf{U}) = 0 \end{aligned} \tag{9}$$

Where the optimum point on the failure surface is called the MPP $\mathbf{U}^*$ and the reliability index is computed $\beta=\|U^*\|$. To find the solution to Eq.(9), the HL-RF algorithm is a popular choice for conducting a reliability analysis in RIA because of its simplicity and efficiency.

The reliability analysis in RIA is to minimize the distance $\|\mathbf{U}\|$ in the standard normal space to the failure surface g(**U**)=0.

The HL–RF method, originally proposed for second-moment reliability analysis and later extended to incorporate distributional information, is currently the most widely used approach for solving structural reliability problems. In this study, the method is referred to as the HL–RF method and is based on the following recursive formulation [16,17].

$$\mathbf{U}_{k+1} = \frac{1}{|\nabla g(\mathbf{U}_k)|^2}[\nabla g(\mathbf{U}_k)\mathbf{U}_k - g(\mathbf{U}_k)]\nabla g(\mathbf{U}_k)^T \tag{10}$$

Previous studies have demonstrated that this approach can be regarded as a particular form of the sequential quadratic programming (SQP) technique, where the Hessian of the Lagrangian is simplified to the identity matrix and the step size is fixed at ξk=1. Compared with alternative algorithms, the HL-RF method is attractive because it requires minimal computational effort and memory per iteration. Empirical evidence further indicates that the method not only converges in most cases but also achieves convergence at a relatively fast rate. Nevertheless, earlier investigations have revealed that under specific circumstances, the algorithm may encounter convergence difficulties. To enhance its stability, several improvements have been suggested. Rackwitz and co-workers [17] introduced two modification strategies: the first redefines the iteration point using U_k_kUk and U_{k+1} from Eq. (10) to ensure the linear constraint is satisfied, while the second applies a two-stage sub-iteration one employing a Newton type search to reach a feasible point, and the other following a tangential search direction to obtain a vector nearly parallel to the gradient.

2.3 Solution of RBF using RIA

The reliability index β is defined with the following optimization model

$$\begin{aligned} &\min && \|\mathbf{U}\| \\ &\text{Subject to} && \tilde{g}(\mathbf{X}(\mathbf{U})) = 0 \end{aligned} \tag{11}$$

The MPP search, there are many general optimization algorithms are applicable for this reliability analysis problem. In this study, we choose the Hasofer Lind and Rackwitz Fiessler (HL-RF) method to perform reliability analysis in RIA because of its simplicity and efficiency.

The first-order reliability analysis in RIA is to minimize the distance $\|\mathbf{U}\|$ in the standard normal space to the failure surface $\tilde{g}(\mathbf{X}(\mathbf{U})) = 0$. Thus, the HL-RF method can be expressed as

$$\mathbf{U}_{k+1} = \frac{\nabla\tilde{g}(\mathbf{U}_k)\{\nabla\tilde{g}(\mathbf{U}_k)\mathbf{U}_k - \tilde{g}(\mathbf{U}_k)\}}{\left\|\nabla\tilde{g}(\mathbf{U}_k)\right\|^2} \tag{12}$$

where $\nabla\tilde{g}(\mathbf{U_k})$ represents the gradient of the approximated state limit function $\tilde{g}(\mathbf{X}(\mathbf{U}))$ at $\mathbf{U}_k$

$$\beta=-\Phi^{-1}(F_{Gi}(0)) \tag{13}$$

where β is the reliability index

2.4 Computational procedures

In this study, radial basis functions (RBFs) are employed to approximate the limit state function in combination with the Latin Hypercube Sampling (LHS) technique. Six types of RBFs are considered, namely Gaussian, Multi-Quadric, Inverse Multi-Quadric, Thin Plate Spline, Cubic, and Linear. The shape parameter α_i is varied systematically to examine its influence on the estimated reliability index and failure probability. The HL-RF algorithm is then applied to evaluate the reliability index and failure probability based on the constructed surrogate model. The overall procedure is outlined in Fig. 1 and can be summarized as follows:

The flowchart of the proposed is illustrated in Fig.1 and the calculation steps of the proposed method can be described as follows

Step 1: Select random variables **X** and define the state function g(**X**) according to the engineering problem.

Step 2: Define the sampling space and generate sampling points by using Latin hypercube sampling method.

Step 3: Conduct FEM analysis at each correponding samples and compute the corresponding value of the performance function.

Step 4: Construct radial basis function approximate model and calculate the weighting coefficient $\boldsymbol{\lambda}$ vector using Eq.(3).

Step 5: RBF response approximate model.

Step 6: Determine the MPP using RIA for RBF approximate model. Compute the distance β to this new design point from the origin, probability of failure by HL-RF algorithm

Step 7: Check the convergence criterion.

Step 8: Repeat step 2 until satisfaction of the convergence criterion

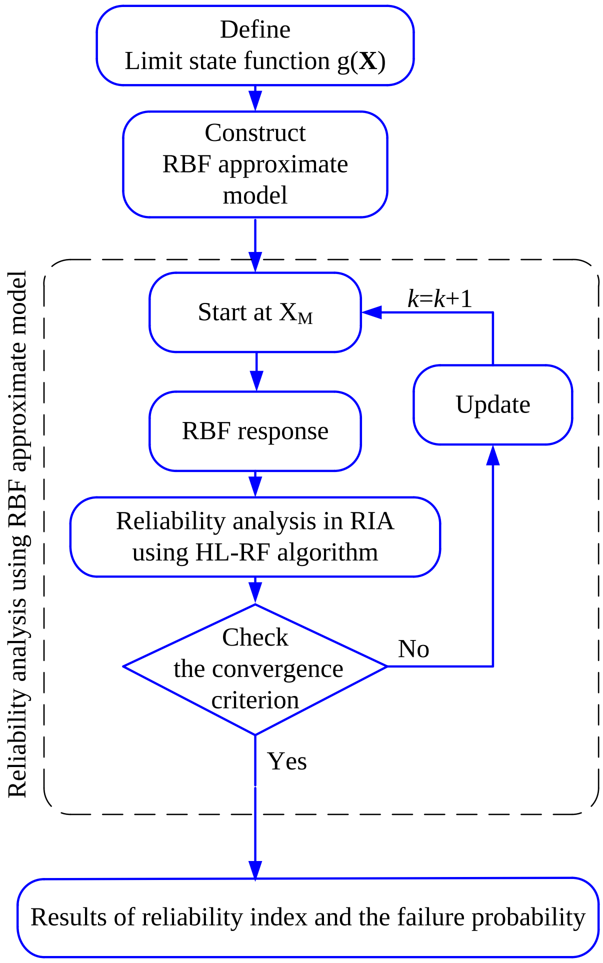

Figure 1: Flowchart of RBF-based reliability index approach

3. RESULTS AND DISCUSSION

3.1 A frame structure

In this case study, a 12-story, 3-bay frame structure is considered, as illustrated in Fig. 2. The cross-sectional areas A_i and the horizontal load P are modeled as independent random variables, with their statistical properties summarized in Table 1. The corresponding sectional moments of inertia are expressed as:

$I_i=\alpha_i A_i^2$ ($\alpha_1=\alpha_2=\alpha_3=0.0833$, $\alpha_4=0.2667$, $\alpha_5=0.2$). The Young's modulus is treated as deterministic $E=2\times10^7 kN/m^2$.

The section area of truss is independent Lognormal variables with mean $A_1=0.25m^2$, $A_2=0.160m^2$, $A_3=0.360m^2$ $A_4=0.200m^2$ and $A_5=0.150m^2$. The load are independent Type I largest variables with mean P=30,000KN.

The limit state function is defined as

$$g(\mathbf{X})=0.096-\mathbf{U}_{max} \tag{14}$$

Here, $\mathbf{U}_{max}$ represents the maximum horizontal displacement expressed as a function of the basic random variables. The corresponding limit state function is implicit, and the structural response is evaluated through finite element analysis (FEM).

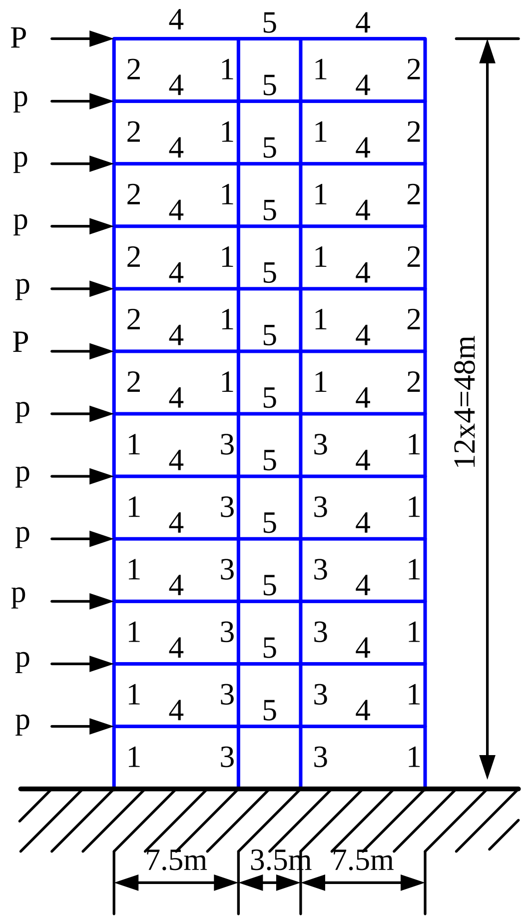

Figure 2: Linear portal frame structure

The results of the proposed method are compared with MCS method with 1000000 samples, probability of failure P_f=7.5058×10^{-2}, and the corresponding reliability index β=1.4391, which are regarded as the exact referenced solution [18] [19] [20] [23].

Table 1: Random properties of portal frame structure

Variable	Mean	Standard deviation	Unit	Distribution
A_1	0.250	0.025	m^2	Lognormal
A_2	0.160	0.016	m^2	Lognormal
A_3	0.360	0.036	m^2	Lognormal
A_4	0.200	0.020	m^2	Lognormal
A_5	0.150	0.015	m^2	Lognormal
P	30,000	7,500	kN	Type I largest

Table 2 shows the results obtained from estimating the probability of failure using the proposed approach for different basis functions as Gaussian, multi-quadric, inverse multi-quadric, thin plate spline, cubic and linear. The observation is that the probability of failure from Gaussian, multi-quadric and inverse multi-quadric very close to the exact solution.

The estimates of the reliability estimates strongly depend on the choice of the different radial basis functions. The results and relative error of the three basis functions are shown by Gaussian, multi-quadric, inverse multi-quadric, thin plate spline, cubic and linear. It can be seen that the analysis results of the Gaussian, multi-quadric and inverse multi-quadric are all very close to the exact ones. Using Gaussian, multi-quadric and inverse multi-quadric can obtain more accurate P_f results than thin plate spline, cubic and linear.

Table 2: Reliability index and failure probabilistic results of frame structure for different radial basis functions

Basis functions	Probability of failure	Reliability index		NFE
		Reliability	error	
Gaussian	7.460×10^{-2}(0.610%)	1.442	0.208%	27
Multi-Quadric (MQ)	8.065×10^{-2}(7.450%)	1.401	2.641%	27
Inverse MQ (IMQ)	7.446×10^{-2}(0.797%)	1.443	0.278%	27
Thin Plate Spline (TPS)	12.070×10^{-2}(60.809%)	1.171	18.624%	30
Cubic	3.220×10^{-2}(57.099%)	1.849	28.492%	17
Linear	9.440×10^{-2}(25.769%)	1.314	8.687%	27

Comparison is made between the proposed method and the MCS in terms of accuracy and efficiency as shown in Table 2. It can be found that the thin plate spline, cubic and linear shows a 60.809%, 57.099%, 25.769% deviation for the probability of failure and 18.624%, 28.492%, 8.687% error for the reliability index whereas Gaussian, multi-quadric and inverse multi-quadric only 0.610%, 7.450%, 0.797% and 0.208%, 2.641%, 0.278% respectively, which indicates that the Gaussian, multi-quadric and inverse multi-quadric is more accurate (Fig.3).

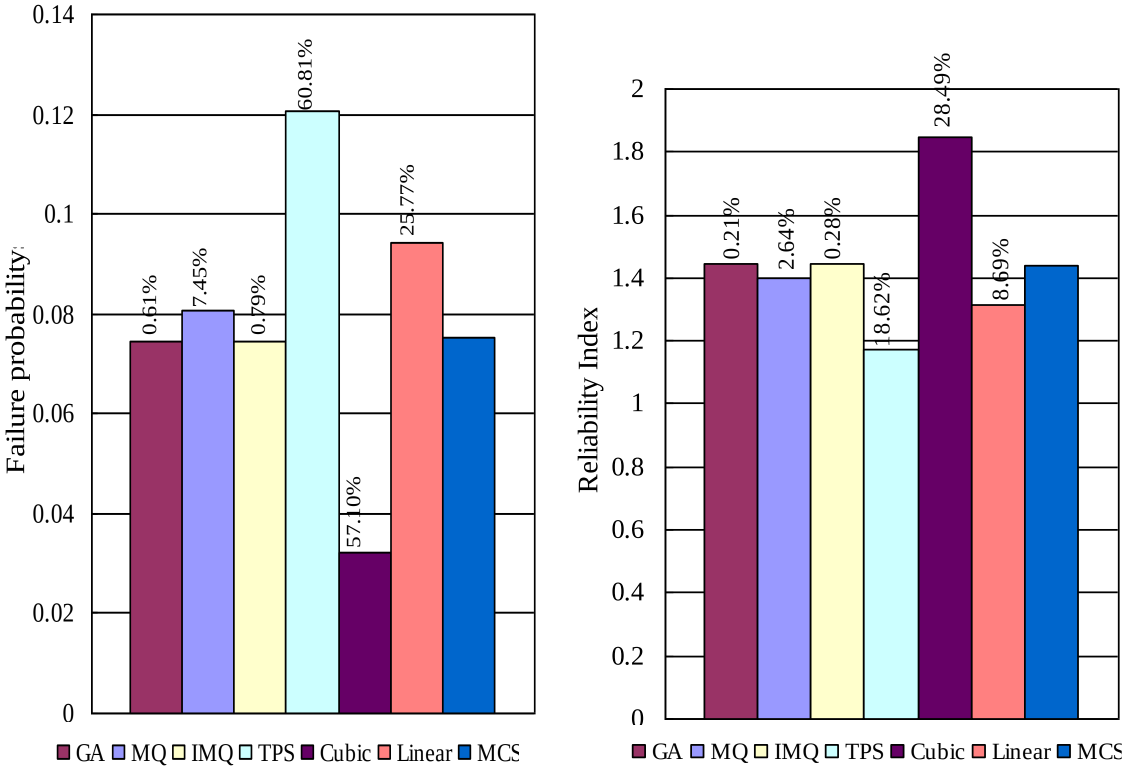

Figure 3: Failure probabilistic and Reliability index results of frame structure

Table 2 summarizes the reliability index and failure probability estimates obtained from the response surface method for different values of the shape parameter α_i when applying Gaussian, Multi-Quadric, and Inverse Multi-Quadric functions. A key observation is that the results from Gaussian and Multi-Quadric with $\alpha_i \approx 0.001$, as well as those from Inverse Multi-Quadric with $\alpha_i \approx 0.25$, yield probabilities that remain nearly unchanged and closely match the reference values. Notably, the Gaussian function shows significant sensitivity to variations in α_i. For very small values of α_i, such as 0.001, it slightly overestimates the failure probability but remains close to the Monte Carlo Simulation (MCS) results. As α_i increases, the estimates initially approach the exact solution but eventually become larger than those from MCS. These trends are consistent with the observations in Fig.3, although the actual limit-state surface considered is more nonlinear and slightly asymmetric compared to the idealized case illustrated.

3.2 A three bay five storey rigid frame structure

In this example, a three bay five storey rigid frame structure [21] [22] [20] [24] is used as the fifth example. This structure, which is shown in Fig.4, is characterized by 21 random variables. The statistical properties and the structural data are given in Table 3 and Table 4, respectively.

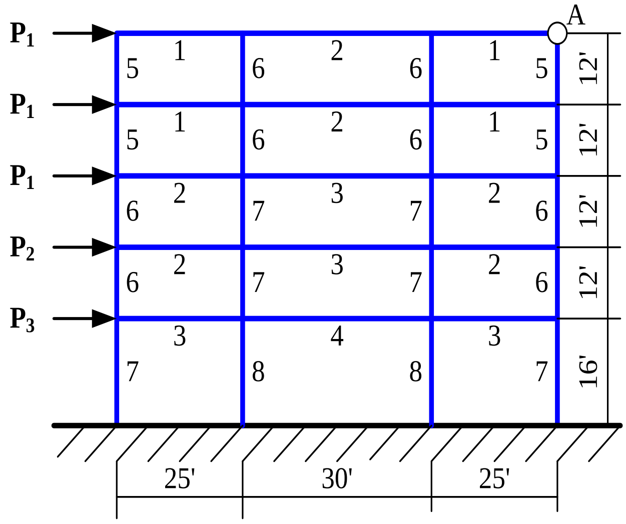

Figure 4: Three bay five storey rigid frame structure

Table 3: Properties of three bay five storey rigid frame structure

Element	Young's modulus	Moment of inertia	Cross section area
1	E_1	I_5	A_5
2	E_1	I_6	A_6
3	E_1	I_7	A_7
4	E_1	I_8	A_8
5	E_2	I_1	A_1
6	E_2	I_2	A_2
7	E_2	I_3	A_3
8	E_2	I_4	A_4

Some of the basic variables are assumed to be correlated as follows. All cross sectional properties are correlated as $\rho_{AiAj} = \rho_{IiIj} = \rho_{IiAj}$ =0.13. All loadings are correlated by ρ=0.5, while the cross sectional area and the moment of inertia of each element type are correlated by ρ=0.95. The two different modulus of elasticity E_1 and E_2 are correlated by ρ=0.9 and all other variables are summed to be uncorrelated.

Table 4: Statistical properties of random variables for three bay five storey rigid frame structure

Variable	Mean	Standard deviation	Unit	Distribution
P_1	30	9	kips	Rayleigh
P2	20	8	kips	Rayleigh
P3	16	6.4	kips	Rayleigh
E1	454,000	40,000	kips /ft^2	Normal
E2	497,000	40,000	kips /ft^2	Normal
I_1	0.94	0.12	ft^4	Normal
I_2	1.33	0.15	ft^4	Normal
I_3	2.47	0.30	ft^4	Normal
I_4	3.00	0.35	ft^4	Normal
I_5	1.25	0.30	ft^4	Normal
I_6	1.63	0.40	ft^4	Normal
I_7	2.69	0.65	ft^4	Normal
I_8	3.00	0.75	ft^4	Normal
A_1	3.36	0.60	ft^2	Normal
A_2	4.00	0.80	ft^2	Normal
A_3	5.44	1.00	ft^2	Normal

A_4	6.00	1.20	ft^2	Normal
A_5	2.72	1.00	ft^2	Normal
A_6	3.13	1.10	ft^2	Normal
A_7	4.01	1.30	ft^2	Normal
A_8	4.50	1.45	ft^2	Normal

The failure criterion is formulated as an implicit limit state function

$$g(\mathbf{X})=0.2-u(\mathbf{X}) \qquad (15)$$

where u(**X**) denotes the actual horizontal displacement as a function of all basic variables. The limit state function is also implicit function, and the structural response is computed by using the FEM.

In the following, the resulting from the proposed method (Table 5) are compared with that of the exact result from MCS with failure probability P_f=2.91x10^{-4}.

Table 5 presents the estimated probabilities of failure obtained using the response surface approach with different radial basis functions, including Gaussian, multiquadric, inverse multiquadric, thin-plate spline, cubic, and linear functions. Notably, the estimates derived from the Gaussian, multiquadric, and inverse multiquadric functions are in very close agreement with the exact solution.

Table 5: Reliability index and failure probabilistic results of three bay five storey rigid frame structure for different radial basis functions

Basis functions	Probability of failure	Reliability index		NFE
		Reliability	error	
Gaussian	2.598×10^{-4}	3.470	0.872%	19
Multi-Quadric (MQ)	35.660×10^{-4}	2.691	21.773%	19
Inverse MQ (IMQ)	5.440×10^{-4}	3.267	5.029%	19
Thin Plate Spline (TPS)	0.175	0.934	72.849%	19
Cubic	0.304	0.514	85.058%	19
Linear	331.194×10^{-4}	1.837	46.599%	19

The reliability estimates are strongly influenced by the choice of radial basis functions. Table 5 presents the estimated results and relative errors obtained using the Gaussian, multiquadric, and inverse multiquadric basis functions. The results indicate that all three functions yield estimates that are in close agreement with the exact solutions. Furthermore, the Gaussian, multiquadric, and inverse multiquadric functions provide more accurate failure probability (P_f) estimates than the thin-plate spline, cubic, and linear functions. A comparison between the proposed method and Monte Carlo simulation (MCS) in terms of accuracy and computational efficiency is also conducted.

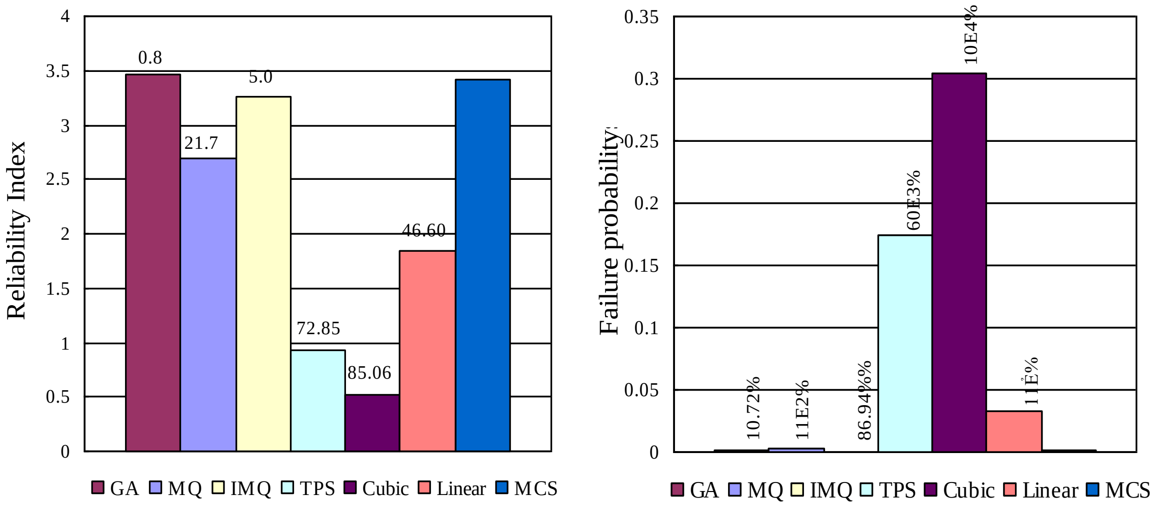

Figure 5: Reliability index and failure probabilistic results of three bay five storey rigid frame structure for different radial basis functions

In Fig.5 is shown that the Gaussian shows a 0.872% deviation, multi-quadric shows a 21.773% deviation, inverse multi-quadric shows a 5.029% deviation, thin plate spline shows a 72.849% deviation, cubic shows a 85.058% deviation, Linear shows a 46.599% deviation for the reliability index, which indicates that the Gaussian, multi-quadric and inverse multi-quadric is more accurate.

Table 5 shows the results obtained from estimating the probability of failure using the response surface approach for different values of different parameters α_i of radial basis functions as Gaussian, MQ and IMQ.

The observation from Fig.5 is that at α_i about 0÷0.4 the probability estimated from Gaussian and at α_i about 0.01 the probability estimated very close to the exact ones. The probability estimated from Multi-Quadric and Inverse Multi-Quadric for small changes and the probability estimated from Gaussian for large changes at α_i about 0.4÷1.

4. CONCLUSIONS

Several studies have examined the role of basis functions and their parameters in the response surface methodology for reliability analysis. Commonly used radial basis functions (RBFs) include Gaussian, Multi-Quadric, Inverse Multi-Quadric, Thin Plate Spline, Cubic, and Linear forms. In such formulations, the shape parameter α_i is often varied to assess its influence on the estimated probability of failure and reliability index. Numerical investigations indicate that RBFs can significantly affect the accuracy of the surrogate model. In particular, results obtained with Gaussian, Multi-Quadric, and Inverse Multi-Quadric functions closely match those from Monte Carlo simulations when appropriate values of α_i are selected. For instance, in the tested examples, Gaussian RBFs provided solutions comparable to Monte Carlo outcomes when α_i ranged from 0.01 to 0.3. Nonetheless, the optimal choice of parameter depends strongly on the characteristics of the limit state function, and systematic guidelines remain scarce in the literature. The present study seeks to address this gap by providing further insights into the effects of RBF selection and parameter tuning on structural reliability estimation.

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NGHIÊN CỨU ẢNH HƯỞNG CỦA CÁC HÀM CƠ SỞ ĐẾN ƯỚC TÍNH ĐỘ TIN CẬY CỦA KẾT CẤU SỬ DỤNG HÀM CƠ SỞ BÁN KÍNH

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Tóm tắt. Phương pháp Monte Carlo và phương pháp phần tử hữu hạn cho phân tích độ tin cậy kết cấu thường dẫn đến chi phí tính toán quá cao. Trong ước tính độ tin cậy của các kết cấu phức tạp, phương pháp tiếp cận bề mặt đáp ứng dựa trên RBF đã được đề xuất như một cách để ước tính hàm trạng thái giới hạn ngầm định. Tuy nhiên, các tham số và hàm cơ sở của RBF ảnh hưởng đến kết quả phân tích độ tin cậy kết cấu nhưng không có hướng dẫn về cách chọn giá trị phù hợp cho các tham số và hàm cơ sở. Do đó, nghiên cứu này nghiên cứu ảnh hưởng của các tham số và hàm cơ sở đến ước tính độ tin cậy kết cấu dựa trên RIA bằng cách sử dụng các hàm cơ sở xuyên tâm (RBF) như Gaussian (GA), Multi-Quadric (MQ), Inverse Multi-Quadric (IMQ), Thin Plate Spline (TPS), Cubic và Linear. RBF được sử dụng để xấp xỉ các hàm trạng thái giới hạn kết hợp với chiến lược Lấy mẫu siêu khối Latin (LHS). Thuật toán HL-RF được áp dụng để thu được chỉ số độ tin cậy và xác suất phá hoại dựa trên mô hình RS đã xây dựng.

Từ khóa: Phân tích độ tin cậy kết cấu; phương pháp độ tin cậy bậc nhất; phương pháp bề mặt đáp ứng; hàm cơ sở bán kính.

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