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PHYSICS-INFORMED DIFFERENTIAL EVOLUTION FOR NONLINEAR ANALYSIS OF CABLE NETS

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Abstract. In this article, a physics-informed differential evolution approach is developed to estimate the nonlinear behaviors of pretensioned cable structures without using any structural analyses. Rather than solving nonlinear equations through conventional numerical methods, this approach employs differential evolution to minimize Total Potential Energy (TPE). Therein, the TPE is designed as an objective function to guide the searching process of the Differential Evolution (DE) algorithm. Once the minimum TPE value is found, the nonlinear behavior of structures can be easily obtained. Three benchmarks are examined to determine the efficiency of the suggested framework for geometrically nonlinear analysis of cable structures. The results demonstrate that the proposed approach is easy to implement and delivers high accuracy.

Keywords. Nonlinear analysis; Cable net structure; Geometric nonlinearity; Differential evolution

1 INTRODUCTION

Cable members are frequently employed as important components in tension structures due to their significant structural advantages, especially in designs that require high strength, light weight, flexibility, and cost-effectiveness. Cable-supported bridges, power lines, and large-span roof structures are examples of such applications. However, this structure can lead to instability because of their significant geometric nonlinearity. As a result, when studying the cable net structures, the impact of large deflections must be carefully considered. And this presents a significant challenge that has garnered the attention of many researchers

In general, many different techniques for analyzing cable structures have been presented. And these works were commonly classified into two classes called the stiffness-based and energy methods. Therein, the stiffness matrix, which was known as the first approach, was iteratively updated as the structure deforms due to changes in material and geometry properties. There were two alternative approaches developed to build the stiffness matrix of the cable elements. The first one was based on the interpolation function, and the other one relied on analytical expressions. The finite element method, which was known as the first sub-method, approximates the cable element by using interpolation polynomial functions. And this approach has been effectively used to estimate the nonlinear behaviors of cables. For instance, Knudson [1] developed a two-node element for solving static and dynamic analysis of cable-net structures. To enhance the accuracy, a multi-node element was introduced by Chen [2]. The two-node element was suitable for cables with high pretension and small sag, while the multi-node element was employed for cables with larger sag [3]. The second sub-approach formulated a two-node elastic catenary cable element using exact analytical expressions to precisely capture the behavior of cables [4, 5]. This strategy resulted in greater accuracy and reduced the number of degrees of freedom needed for cable structures, compared to the first sub-method. Despite the considerable success of stiffness-based algorithms, their limitations persisted due to the nonlinear incremental-iterative methods. To address these limitations, optimization algorithms were used to minimize energy functions. Therein, the gradient based method has been applied in the structural analysis. For example, Sufian [6] introduced an entropy-based optimization algorithm to find the minimum total potential energy stored in the structure. In addition, Lewis [7] studied the relative efficiency of dynamic relaxation and stiffness matrix methods to calculate the nonlinear static response of frame and pretensioned cable structures. Besides, a minimum principle of complementary energy was suggested by Kanno and Ohsaki [8] for considering the prestressed cable nets with geometrical nonlinearities and nonlinear elastic materials. While gradient-based frameworks achieved rapid convergence in just a few iterations, the derivative information was required to search for the solution. As an effective alternative to address the above challenge, the gradient-free algorithms can find near-optimal solutions without the need for gradient knowledge. And geometric nonlinearity of the cable structures was exploited using Harmony Search by

Toklu [9, 10]. Because of the above-mentioned advantages of metaheuristic methods, the DE algorithm has proven effective in tackling complex optimization problems. However, until now, it has not been used for the nonlinear analysis of cable systems so far.

In this paper, a physics-informed differential evolution framework is introduced to analyze the cable structures. Therein, the TPE, which is known as an objective function, is minimized to estimate the geometrically nonlinear behaviors. The structural responses are obtained immediately after the optimization procedure is completed, without requiring any structural analysis. Several numerical examples for analyzing the geometric nonlinearity of cable net systems are examined to demonstrate the efficiency of the suggested approach. The obtained results showed that the proposed approach is not only simple in procedure, but also yields high accuracy.

The remaining paper is organized as follows. Section 2 presents the formulation of the TPE of the cable structure as an optimization problem. Next, Section 3 provides the DE algorithm. And several cable structures are performed in Section 4 to show the effectiveness and accuracy of our method. Finally, Section 5 draws conclusions from the obtained results of the benchmark problems.

2 FORMULATION OF THE PROBLEM

A cable structure is defined as a unique configuration of straight links possessing extensional stiffness. The external forces are applied at the nodal intersections of cables. A cable net structure with the initial pretension forces consisting of *n* joints and *m* cable members is considered. And its TPE is defined by summing the external work and strain energy, as follows [10]:

$$\Pi_p = U + W, \tag{1}$$

where

$$U = \sum_{k=1}^{m}\left[U_{0,k} + f_{0,k}e_k + \frac{E_k A_k}{2\ell_{l_0,k}} e_k^2\right], \tag{2}$$

$$W = -\mathbf{f}^T\hat{\mathbf{d}}, \tag{3}$$

$$U_{0,k} = \frac{E_k A_k}{2\ell_{0,k}} e_{t_0,k}^2, \tag{4}$$

in which $\Pi_p, U,$ and W are the total potential energy, strain energy, and external work, respectively; E_k, A_k, and $U_{0,k}$ refer to the Young's modulus, cross-sectional area, and the initial strain energy of the *k*th cable member when subjected to the pretension force; $\ell_{0,k}, \ell_{l_0,k}$, and e_k denote the length without and with pretension force, and elongations resulting from the pretension force of the *k*th member; $\mathbf{f}$ and $\hat{\mathbf{d}}$ are the external force and displacement vectors.

A cable *k*th element in space with ends coordinates (x_i, y_i, z_i) and (x_j, y_j, z_j), as shown in Figure 1, is considered to build the formulation of the elongations. Therein, $\ell_{0,k}$ and $\ell_{l_0,k}$ are defined the length of the cable element before and after the application of the pretension force, respectively. They are expressed as follows:

$$\ell_{l_0,k} = \sqrt{\ell_{0x,k}^2 + \ell_{0y,k}^2 + \ell_{0z,k}^2}, \tag{5}$$

$$\ell_{0,k} = \frac{\ell_{l_0,k} E_k A_k}{f_{0,k} + E_k A_k}, \tag{6}$$

in which $\ell_{0x,k} = x_j - x_i$; $\ell_{0y,k} = y_j - y_i$; $\ell_{0z,k} = z_j - z_i$; $f_{0,k}$ denotes the initial pretension force in the *k*th cable member.

When the external loads are applied to the cable structure, the new length $\ell_{f,k}$ corresponding to the displacement field is given by:

$$\ell_{f,k} = \sqrt{\ell_{fx,k}^2 + \ell_{fy,k}^2 + \ell_{fz,k}^2}, \quad (7)$$

where $\ell_{fx,k} = \ell_{0x,k} + \hat{u}_j - \hat{u}_i$; $\ell_{fy,k} = \ell_{0y,k} + \hat{v}_j - \hat{v}_i$; $\ell_{fz,k} = \ell_{0z,k} + \hat{w}_j - \hat{w}_i$; $\hat{u}, \hat{v}$, and $\hat{w}$ represent deflections in the x, y, and z directions at the nodal intersections.

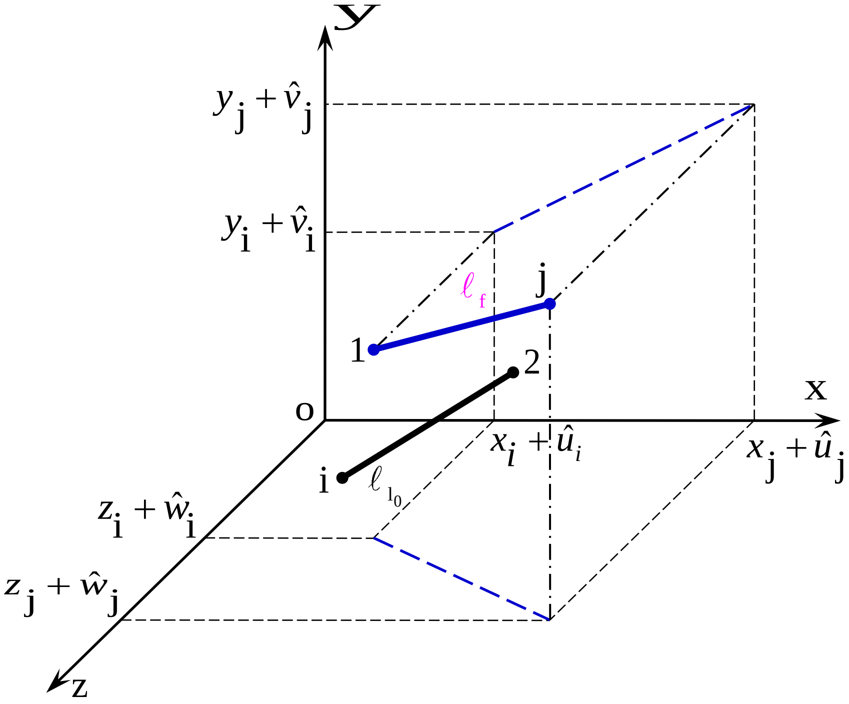

Figure 1: Deformation of a pretensioned cable member.

It is easily be seen that the elongations before and after applying the external loads can be easily determined, as follows:

$$e_{l_0,k} = \ell_{l_0,k} - \ell_{0,k}, \quad (8)$$

$$e_k = \ell_{f,k} - \ell_{l_0,k}. \quad (9)$$

Once the elongations are found, the TPE can be calculated by integrating external work and strain energy. With the deflections as design variables, the TPE serves as an objective function which is minimized by the DE algorithm.

3 DIFFERENTIAL EVOLUTION

Price and Storn [11] introduced the DE, which has been successfully proved as effective for solving optimization problems. And it is utilized as an optimizer to minimize the TPE of the cable structure in this work. The following is a summary of the basic steps in the DE algorithm.

- Initialization

  Initially, a random population of *np* individuals is generated from the search space. A vector with *m* design variables $\hat{\mathbf{d}}_i = \{\hat{d}_{i,1}, \hat{d}_{i,2}, ..., \hat{d}_{i,m}\}$ is the *i*th individual which is defined by

$$\hat{d}_{i,j} = \hat{d}_{min,j} + rand_{i,j}[0,1]\left(\hat{d}_{max,j} - \hat{d}_{min,j}\right) \quad i = 1, 2, ..., np; \; j = 1, 2, ..., m, \quad (10)$$

  which $\hat{d}_{min,j}$ and $\hat{d}_{max,j}$ are the lower and upper bounds of $\hat{d}_j$; $rand_{i,j}[0,1]$ is a random number uniformly distributed over the interval. In this study, the design variables are the displacement field.

- Mutation

Next, a mutation strategy associated with a target vector $\mathbf{x}_i$ is applied to generate a mutant vector $\mathbf{v}_i$. There are four popular mutation operations. They are given by

$$\text{best}/1\ \ \mathbf{v}_i = \hat{\mathbf{d}}_{best} + F\left(\hat{\mathbf{d}}_{r_1} - \hat{\mathbf{d}}_{r_2}\right), \quad (11)$$

$$\text{best}/2\ \ \mathbf{v}_i = \hat{\mathbf{d}}_{best} + F\left(\hat{\mathbf{d}}_{r_1} - \hat{\mathbf{d}}_{r_2}\right) + F\left(\hat{\mathbf{d}}_{r_3} - \hat{\mathbf{d}}_{r_4}\right), \quad (12)$$

$$\text{rand}/1\ \ \mathbf{v}_i = \hat{\mathbf{d}}_{r_1} + F\left(\hat{\mathbf{d}}_{r_2} - \hat{\mathbf{d}}_{r_3}\right), \quad (11)$$

$$\text{rand}/2\ \ \mathbf{v}_i = \hat{\mathbf{d}}_{r_1} + F\left(\hat{\mathbf{d}}_{r_2} - \hat{\mathbf{d}}_{r_3}\right) + F\left(\hat{\mathbf{d}}_{r_4} - \hat{\mathbf{d}}_{r_5}\right), \quad (12)$$

where r_1, r_2, r_3, r_4, and r_5 are integer numbers which are selected from $\{1,....,i-1,\ i+1,...,np\}$; F denotes the scale factor which is randomly picked up from (0, 1]; and $\hat{d}_{best}$ is the best individual in the population.

Accordingly, the mutant vector $\mathbf{v}_i$ can be violated its boundary constraint. Therefore, it needs to be modified to the allowable limits of the variable designs, as follows:

$$v_{i,j} = \begin{cases} 2\hat{d}_{min,j} - v_{i,j} & \text{if } \ v_{i,j} < \hat{d}_{min,j}, \\ 2\hat{d}_{max,j} - v_{i,j} & \text{if } \ v_{i,j} > \hat{d}_{max,j}, \\ v_{i,j} & \text{otherwise.} \end{cases} \quad (13)$$

- Crossover

Thirdly, a crossover scheme is employed to enhance the diversity of individual vectors in the existing population. Accordingly, a trial vector $\mathbf{u}_i$ is produced by the mutant vector $\mathbf{v}_i$ and its target vector $\hat{\mathbf{d}}_i$. And it is given by

$$u_{i,j} = \begin{cases} v_{i,j} & \text{if } \ j = K \ \text{ or rand } [0,1] \le Cr, \\ \hat{d}_{i,j} & \text{otherwise,} \end{cases} \quad (14)$$

where Cr denotes the crossover control parameter, and K is a randomly selected integer within the range [1, *np*].

- Selection

Finally, a selection scheme is applied to choose the better individuals by comparing the objective function values of the trial vectors $f\left(\mathbf{u}_i\right)$ and target vector $f\left(\hat{\mathbf{d}}_i\right)$ in the present population. This scheme is given by

$$\hat{\mathbf{d}}_i = \begin{cases} \mathbf{u}_i & \text{if } \ f(\mathbf{u}_i) < f(\hat{\mathbf{d}}_i) \\ \hat{\mathbf{d}}_{i,j} & \text{otherwise.} \end{cases} \quad (15)$$

4 NUMERICAL EXAMPLES

In this section, three numerical examples are carried out to illustrate the effectiveness of the DE algorithm in analyzing the cable structures. For verification, the obtained results will be compared with the Backtracking Search optimization Algorithm (BSA) and the Teaching-Learning-Based Optimization (TLBO) for minimizing TPE. In this study, the parameters of the DE are set as follows: crossover factor Cr = 0.9, a maximum of 9000 evaluations, a population size of 40, mutation factor F = 0.8, and the stopping criterion is set to 10^{-6}. Because of its stochastic nature, the final solution of each problem is found after ten runs. To allow a fair comparison of the various techniques, all problems were run on a desktop computer with MATLAB.

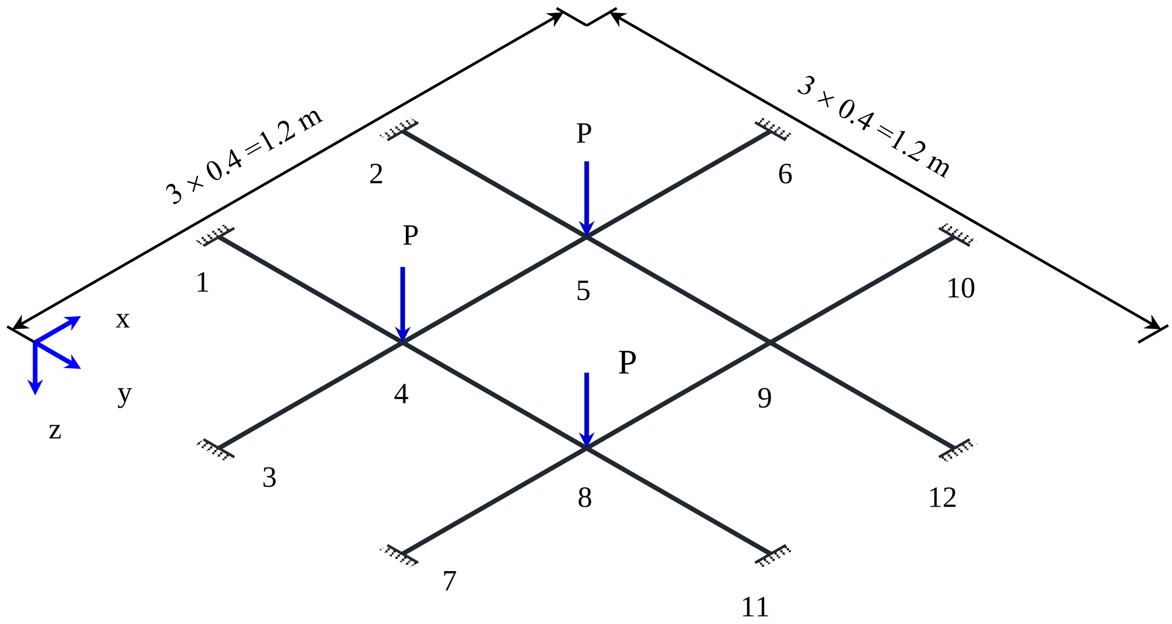

Figure 2: Flat cable net 2 × 2

4. 1 Flat cable net

A flat cable net is considered as the first problem for the nonlinear analyses of cable net, as shown in Figure 2. In this benchmark, all cable elements are the same Young's modulus, cross-sectional area, and pretension force of 124.8 kN/mm^2, 0.785 mm^2, and 200 N, respectively. It is a concentrated force of P = 15 N is applied at free joints 4, 5, and 8 of the system. Several scholars have previously solved this cable structure using different algorithms, such as Toklu [9], Halvordson [12], and Kwan [13].

A comparison of the results obtained by the DE and the other algorithms is summarized in Table 1. In this example, the DE obtains the best TPE (704.8130 N.mm). In addition, it (5880) requires less evaluations than the BSA (25120), and TLBO (10960). Clearly, the DE outperforms existing algorithms both the solution quality and computational cost. Besides, it is simplicity in performance without using any structural analysis as well as the information gradient. The TPE convergence histories obtained by the DE, BSA, and TLBO are shown in Figure 3. It can be easily seen that the proposed algorithm converges very fast in the early iterations, requiring only 5,880 iterations. In contrast, to achieve the optimal solution, the BSA and TLBO used 25120 and 10960, respectively.

Table 1: Comparison of the obtained results for the flat cable net

Deflections	Toklu	Halvordson	Kwan	Present		
	[9]	[12]	[13]	BSA	TLBO	DE
u_4	-0.07	-0.07	-0.08	0.071	0.070	-0.071

v_4	-0.07	-0.07	-0.08	0.071	0.071	-0.071
w_4	12.17	12.20	12.17	12.175	12.155	12.172
u_5	0.04	0.04	0.04	-0.042	-0.042	0.041
v_5	-0.08	-0.08	-0.08	0.078	0.076	-0.078
w_5	11.18	11.20	11.18	11.188	11.183	11.184
u_8	-0.08	-0.08	-0.08	0.079	0.077	-0.077
v_8	0.04	0.04	0.05	-0.042	-0.042	0.041
w_8	11.18	11.20	11.18	11.179	11.170	11.186
u_9	-0.04	-0.04	-0.04	0.040	0.038	-0.038
v_9	-0.04	-0.04	-0.04	0.039	0.038	-0.039
w_9	5.59	5.59	5.59	5.595	5.607	5.596
Best Π_p	704.8458	704.8477	704.8925	704.8134	704.8143	704.8130
Worst Π_p	-	-	-	704.8147	704.8157	704.8146
Mean Π_p	-	-	-	704.8141	704.8148	704.8136
No. of Eval	-	-	-	25120	10960	5880

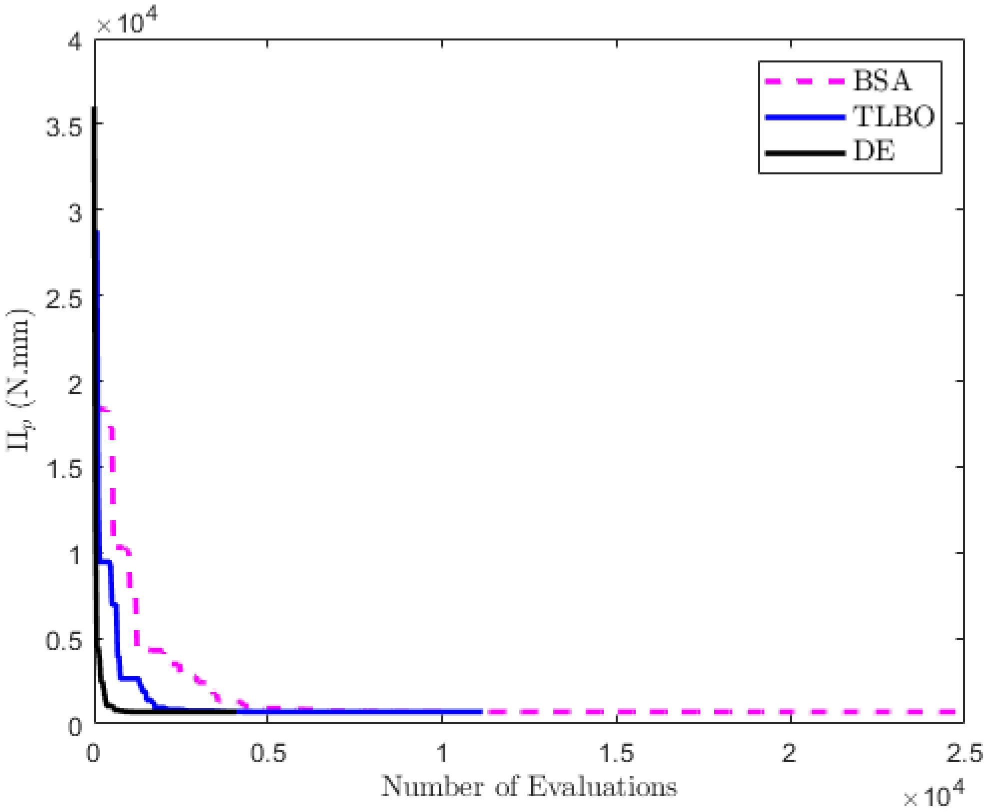

Figure 3: The TPE convergence history of the flat cable net

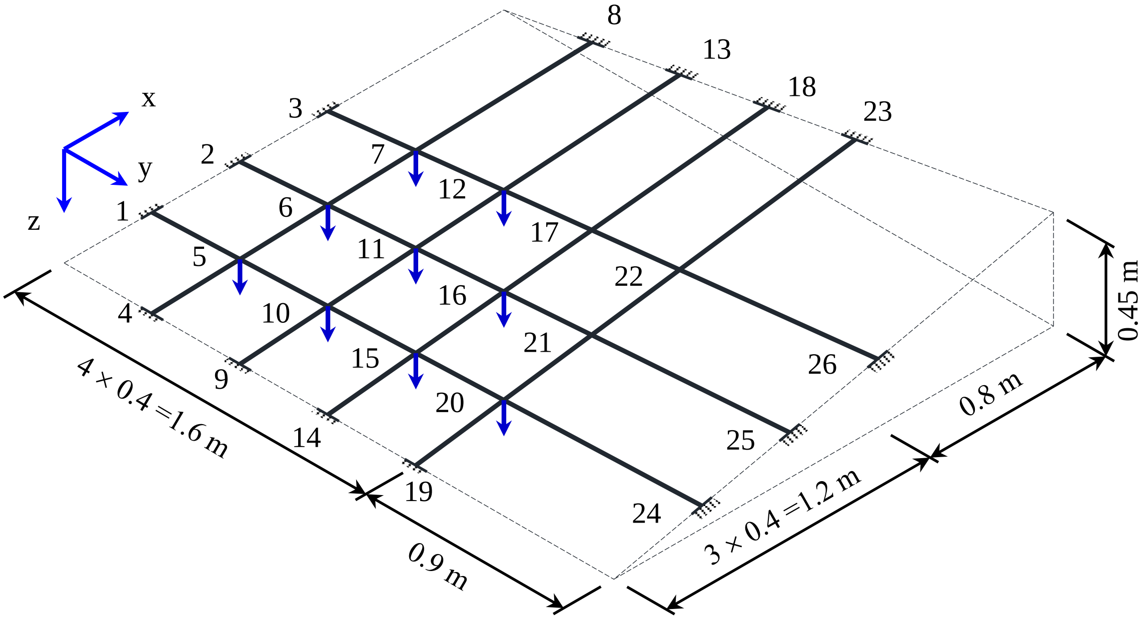

Figure 4: Hyperbolic paraboloid net.

4. 2 Hyperbolic paraboloid net

The second numerical example deals with the hyperbolic paraboloid network shown in Figure 4. This system consists of 31 cables with fourteen constrained joints and other joints allowing free movement. All cable elements have a uniform cross-sectional area of 0.785 mm² and a Young's modulus of 128.3 kN/mm². In addition, to all cable elements to obtain the initial structure geometry, all cable elements get a pretension force of 200 N. A vertically concentrated force of 15.7 N is exerted on the free joints, as shown in Figure 4.

Table 2: Comparison of displacements (mm) for hyperbolic paraboloid net system

Node	Toklu [9]	Kwan [13]	Present BSA	Present TLBO	Present DE
5	19.48	19.52	20.320	18.995	19.349
6	25.59	25.35	26.634	24.663	25.349
7	23.17	23.31	24.308	22.316	23.040
10	25.75	25.86	26.903	24.973	25.601
11	33.86	34.05	35.000	32.720	33.621
12	29.27	29.49	30.929	28.267	29.081
15	25.65	25.79	27.119	24.490	25.357
16	30.96	31.31	32.977	29.487	30.629
17	21.03	21.42	23.384	19.849	20.827
20	21.33	21.48	21.822	19.830	20.567
21	19.67	20.00	21.642	17.736	19.791
22	14.04	14.40	16.248	12.591	14.406
Best Π_p	-	-	1109.475	1091.391	1074.810
Worst Π_p	-	-	1110.051	1091.394	1074.812
Mean Π_p	-	-	1109.822	1091.392	1074.811
No. of Eval.	-	-	360000	198240	142240

As the previously presented flat cable net, Table 2 compares the solutions obtained by the DE and alternative algorithms. It is clear that the DE can accurately estimate the response of cable with the best TPE. In addition, this algorithm demonstrates efficiency compared to various algorithms when the structural cable becomes more complex. Specifically, the DE takes only 142240 evaluation functions, while the BSA and TLBO require 360000 and 198240 to achieve the solution as possible.

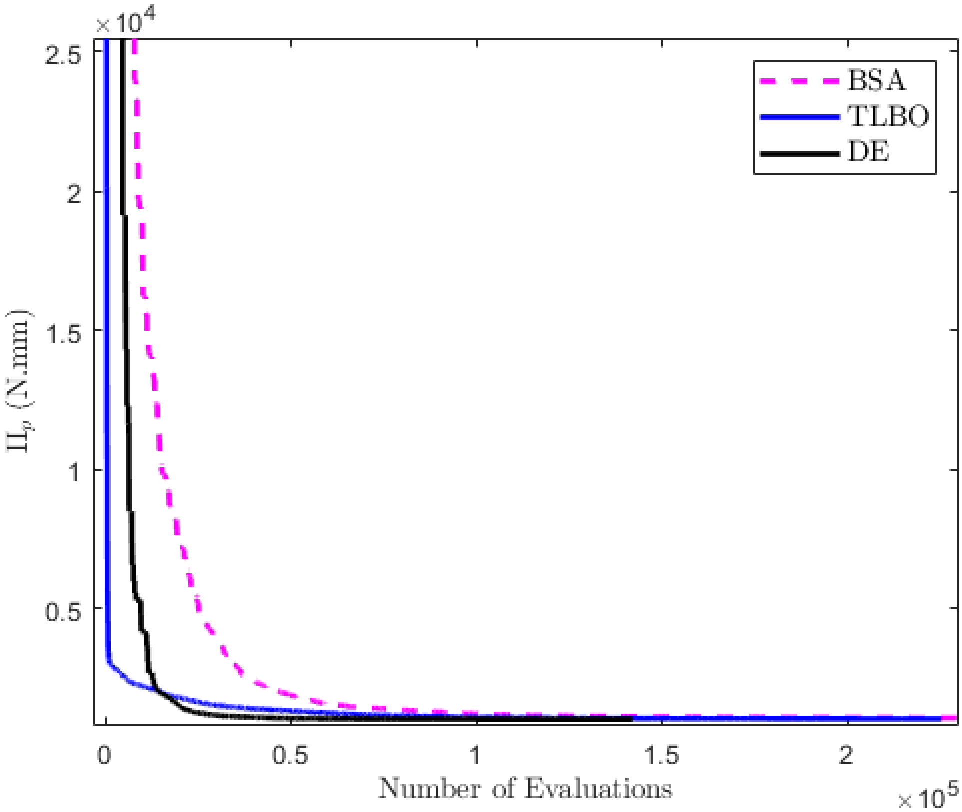

Figure 5: The TPE convergence history of the hyperbolic paraboloid network

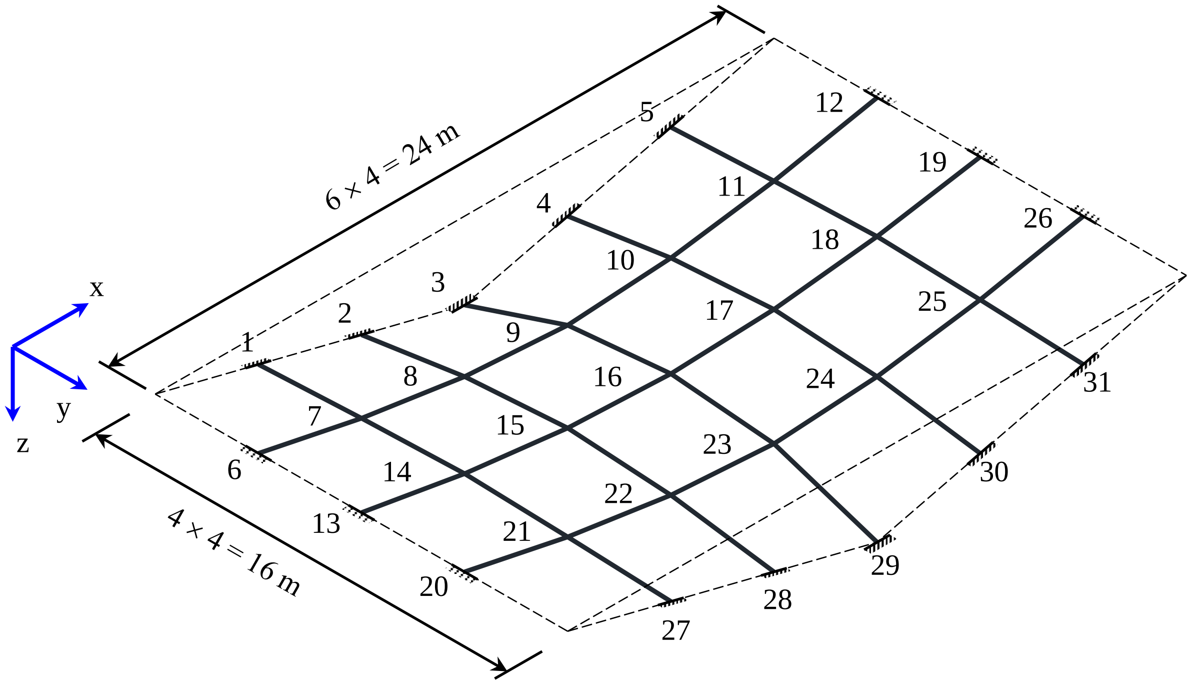

Figure 6: Spatial cable network

4. 3 Spatial cable network

A spatial cable structure, as depicted in Figure 6, is investigated as the last problem. This cable system shows mirror symmetry across both of its central axes. And it was examined by Toklu [9] and Thai [14]. The cable net's initial geometry is configured by applying pretension forces of 90 kN and 30 kN to the cables along the x- and y-axes, respectively. Young's modulus is set to all cables. In this case, the elements in the x- and y-directions have cross-sectional areas of 350 mm² and 120 mm², respectively. Otherwise, a vertical concentrated load of 6.8 kN is applied to all free joints.

The displacements obtained by the DE and other works are reported in Table 3. As expected, the DE found the smallest TPE value (6504279 N.mm) when compared to the BSA (6504295 N.mm) and TLBO (6504288 N.mm). Hence, our approach enhances the accuracy of the structural behaviors. Besides, it can be seen that the deflections found by the DE are close to the previously works with an inaccuracy of less than 1%. In addition, Figure 7 shows the convergence histories found by various algorithms. Clearly, the presented framework converges very quickly in the initial iterations. It requires the number of evaluation functions with only 63500, while the BSA requires a larger number of evaluations 351700.

Table 3: Comparison of deflections (mm) for spatial net structure

Deflections	Lewis [7]	Thai [14]	Toklu [9]	Present BSA	Present TLBO	Present DE
u_7	-5.14	-5.03	-5.03	-5.130	-5.040	-5.030
v_7	0.42	0.41	0.40	0.400	0.399	0.398
w_7	30.41	29.86	29.46	29.461	29.452	29.451
u_8	-2.26	-2.23	-2.22	-2.215	-2.235	-2.225
v_8	0.47	0.46	0.39	0.400	0.395	0.393
w_8	17.70	17.29	17.08	17.978	17.100	17.098
u_9	0	0	0	0.000	0.000	0.000
v_9	-2.27	-2.31	-3.12	-2.368	-2.366	-2.356
w_9	-3.62	-3.61	-3.19	-3.200	-3.200	-3.199
u_{14}	-4.98	-4.92	-4.92	-4.9528	-4.9336	-4.9207
v_{14}	0	0	0	0.000	0.000	0.000
w_{14}	43.49	42.85	42.84	42.863	42.845	42.828
u_{15}	-2.55	-2.55	-2.55	-2.582	-2.557	-2.548
v_{15}	0	0	0	0.000	0.000	0.000
w_{15}	44.47	44.26	44.27	44.342	44.268	44.242
u_{16}	0	0	0	0.000	0.000	0.000
v_{16}	0	0	0	0.000	0.000	0.000
w_{16}	41.65	42.08	42.08	42.083	42.065	42.055
Best Π_p	6505782	6505527	6505019	6504295	6504288	6504279
Worst Π_p	-	-	-	6504299	6504292	6504281
Mean Π_p	-	-	-	6504297	6504290	6504280
No. of Eval.	-	-	-	351700	60700	63500

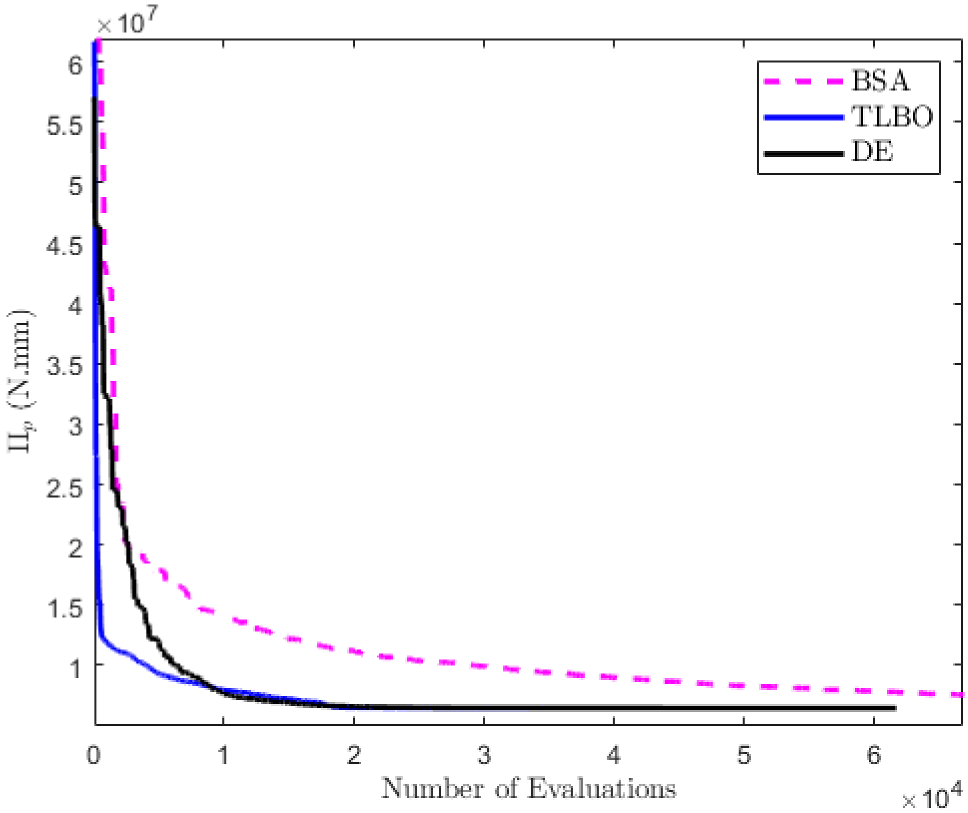

Figure 7: The convergence rate of TPE for the spatial cable network

5 CONCLUSIONS

In this work, a physics-informed differential evolution approach has been developed to address the geometric nonlinearity of cable net structures. To achieve the displacement field, the TPE of the system is considered as an objective function which is minimized by the DE algorithm. The effectiveness and simplicity of the proposed approach are verified through three benchmarks the analysis of cable net structures. The obtained solutions indicated that the DE found the best TPE values and outperformed the other algorithms. One interesting aspect of this approach can find the solution without using any structural analyses. With these outstanding features, it holds significant potential for addressing nonlinear structural analysis challenges without depending on incremental-iterative algorithms.

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THUẬT TOÁN TIẾN HÓA ĐƯỢC THÔNG TIN VẬT LÝ CHO PHÂN TÍCH PHI TUYẾN MẠNG CÁP

MAI TIẾN HẬU

Khoa Cơ khí, Đại học Công nghiệp Thành phố Hồ Chí Minh

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Tóm tắt. Trong nghiên cứu này, một khuôn khổ tiến hóa được thông tin vật lý được phát triển để xác định đáp ứng phi tuyến của mạng cáp không sử dụng phương pháp phần tử hữu hạn. Thay vì giải hệ phương trình phi tuyến như các phương pháp số truyền thống, tiếp cận này sử dụng thuật toán tiến hóa để cực tiểu hàm năng lượng thế năng. Trong đó, hàm năng lượng thế năng được thiết kế như hàm mục tiêu để định hướng quá trình tìm kiếm của thuật toán di truyền. Một khi giá trị cực tiểu hàm năng lượng thế năng được tìm thấy, các đáp ứng phi tuyến của mạng cáp có thể dễ dàng và nhanh chóng đạt được. Một vài ví dụ số được khảo sát để đánh giá hiệu quả của khuôn khổ trình bày cho phân tích phi tuyến hình học cấu trúc mạng cáp. Kết quả đạt được thể hiện khuôn khổ trình bày là đơn giản để thực hiện và đạt được độ chính xác cao.

Từ khóa. Phân tích phi tuyến; Cấu trúc lưới cáp; Tính phi tuyến hình học; Sự phát triển khác biệt.

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