AN EFFICIENT RESPONSE SURFACE TECHNIQUE FOR SENSITIVITY ESTIMATION IN STRUCTURAL RELIABILITY ANALYSIS

MINH QUANG CHAU
Institute of International and Postgraduate Education, Industrial University of Ho Chi Minh City
**Corresponding author: chauminhquang@iuh.edu.vn*
DOIs: https://doi.org/10.46242/jstiuh.v71i5.5057

ABSTRACT. The response surface method (RSM) is a powerful structural reliability method using the values of the function at specific points that approximates the limit state function with a polynomial expression. The analytical function replaces the exact limit state function which the computational time required for the assessment of the reliability of structural systems can be reduced significantly. However, the location of the sample points has been investigated by several authors and the performance of the response surface method is still under discussion. Therefore, this study proposes a new response surface method for sensitivity estimation of parameters in structural reliability analysis. A first oder polynomial without cross terms is adopted to approximate the limit-state function, and the sensitivity vector of the limit state function can be obtained. A experimental design with 4n+1 sampling points includes 2n+1 sampling points are chosen along the coordinate axes of the U-space of standard normal random variables, as in the classic RSM and 2n sampling points is rotated according to the sensitivity vector of the limit state function is built. A quadratic polynomial is adopted to approximate the limit-state function, and the most probable point (MPP) can be obtained by conducting the Hasofer-Lind-Rackwitz-Fiessler (HL-RF) algorithm based on the created response surface (RS). To further improve the precision of reliability analysis, Monte Carlo Simulation (MCS) is conducted on the established polynomial to compute the probability of failure. Three numerical examples are considered to demonstrate the advantages of the proposed method.

Keywords: Reliability analysis, performance measure approach, response surface method, radial basis function, first-order reliability method, finite element method, implicit response function.

I. INTRODUCTION

The reliability analysis has been used more and more widely in structural engineering. For large and complex engineering structures, the reliability represents the capacity of supportment for load and environment. Therefore, how to calculate the reliability effectively has important practical significance.

Zhao et al. [1] proposed a new method, support vector regression based response surface method in reliability analysis. The support vector regression algorithm is employed to construct the equivalent limit state function and FORM/SORM is used in the parameter sensitivity estimation. Wang et al. [2] addressed a approach to system reliability analysis with fuzzy random variables and two developments. The first development is that the Saddlepoint Approximation (SAP)-simulation is extended to conduct reliability analysis accounting for the time-dependent degradation process and fuzzy random variables, and they attempt to give a method to select a proper saddle point. The second development is that two system reliability analysis methods are proposed for different scenarios of reliability modeling processes. Hu and Youn [3] presented an asymmetric dimension-adaptive tensor-product (ADATP) method to resolve the difficulties of existing reliability analysis methods. The their method leverages three ideas: (i) an asymmetric dimension-adaptive scheme to efficiently build the tensorproduct interpolation considering both directional and dimensional importance, (ii) a hierarchical interpolation scheme using either piecewise multi-linear basis functions or cubic Lagrange splines, (iii) a hierarchical surplus as an error indicator to automatically detect the highly nonlinear regions in a random space and adaptively refine the collocation points in these regions. Guan et al. [4] presented an efficient analytical Bayesian method for reliability and system response updating without using simulations. The method includes additional information such as measurement data via Bayesian modeling to reduce estimation uncertainties. Laplace approximation method is used to evaluate Bayesian posterior distributions analytically. An efficient algorithm based on inverse first- order reliability method is developed to evaluate system responses given a reliability index or confidence interval. Allaix and Carbone [5] proposed an improvement of the response surface method. An iterative strategy is used to determine a response surface that is able to fit the limit state function in the neighbourhood of the design point. The locations of the sample points used to evaluate the free parameters of the response surface are chosen according to the importance sensitivity of each random variable. Liu [6] utilized Hermite polynomial chaos expansion to build the stochastic response surface function, and the unknown coefficients of the function can be calculated by probabilistic collocation approach. Then, the geometric method can be used to calculate the structural reliability.

Okasha et al. [7] illustrated an approach for updating the lifetime reliability of aging bridges using monitored strain data obtained from crawl tests. It is proposed to use automated finite element model updating techniques as a tool for updating the resistance parameters of the structure. The results from crawl tests are used to update the finite element model and, in turn, update the lifetime reliability. The original and updated lifetime reliabilities are computed using advanced computational tools.

Kingston et al. [8] used an artificial neural network as a response surface function to efficiently emulate the complex finite element model within a Monte Carlo simulation. To ensure the successful and robust implementation of this approach, a genetic algorithm adaptive sampling method is designed and applied to focus sampling of the implicit limit state function towards the limit state region in which the accuracy of the estimated response is of the greatest importance to the estimated structural reliability.

Jiang et al [9] proposed and create a correlation analysis technique mathematically for the non-probabilistic convex model, and based on it develop an effective method to construct the multidimensional ellipsoids on the uncertainty. A marginal convex model is defined to describe the variation range of each uncertain parameter, and a covariance is defined to represent the correlation degree of two uncertain parameters. For a multidimensional problem, the covariance matrix and correlation matrix can be created through all marginal convex models and covariances, based on which the required ellipsoid on the uncertainty can be conveniently achieved. By combining the correlation analysis technique and the reliability index approach, a non-probabilistic reliability analysis method is also developed for uncertain structures.

Weitao Zhao and Zhiping Qiu [10] proposed the control point of experimental points is constructed. The new center point of experimental points is chosen by using the control point instead of the design point. The control point can guarantee that the center point of experimental points lies exactly on the failure surface and is close to the actual design point.

Yanjun Ou et al [11] explored a response surface method for reliability analysis based on iteratively-reweighted-leastsquare extreme learning machines (IRLS-ELM) in which, highly nonlinear implicit performance functions of structures are approximated by the IRLS-ELM. Monte Carlo simulation is then carried out on the approximate IRLS-ELM for structural reliability analysis.

The main aspect of this paper is discussed in the following: the location of the sample points, the polynomial degree of the response surface and the estimation of the probability of failure. First, a response surface provides a local approximation of the limit state function in the neighbourhood of the sample points. Hence the distance between the sample points determines the region where the approximation is expected to be accurate. Of course, the accuracy depends also on the type of response surface and limit state function.

The sample points in the U-space of the standard normal variables are located in the centre of the experimental plan and along the coordinate axes. The distance between the central point and the other points is denoted by the parameter f. If the value of f is large, the response surface interpolates points in the U-space which are quite distant. Therefore, fluctuations of the limit state function between the points may not be reproduced by the response surface. Conversely, if the value of f is low, the response surface can represent appropriately only a small portion of the LSF.

2. MATERIALS AND METHODS

2.1 Structural reliability formulation

The performance or safety of the structure can be described by limit state functions $g(\mathbf{X}) = 0$, which divide the random variable domain in safety and failure domains. The boundary between safety and failure domains is known as failure surface. The failure probability is given by the following multidimensional integral:

$$P_f = \int_{g(X)<0} f_x(X)dX \quad (1)$$

where $f_x(X)$ is the joint probability density function of the random variables. It can be solved directly, via using approximate solutions, or Monte Carlo simulation.

The most popular approximate solution is the First Order Reliability Method (FORM). In this solution, the vector of random variables X and the limit state functions are mapped to the standard normal space U, in a transformation that involves rotation of coordinates and evaluation of equivalent normal distributions. In the space of U the joint probability density function $f_u(U)$ presents radial symmetry, and the most probable failure point, or design point U can be found by solving the following optimization problem:

$$\begin{aligned} &\textit{Which} \ \min \quad \beta = \sqrt{U^T U} \\ &\textit{Subject} \ \text{to} \quad g(U) = 0 \end{aligned} \quad (2)$$

The minimal distance between design point U^* and the origin of U space is known as Reliability Index β. A linearization of the limit state function at the design point yields the first order approximation of the failure probability:

$$P_f = \Phi(-\beta) \quad (3)$$

where U is the standard normal cumulative distribution function.

The problem in Eq. (2) can be solved using HLRF algorithm. When failure probabilities are small, solution of the reliability problem by FORM, Eqs. (2) and (3), represents enormous savings in computing time, as compared with Monte Carlo simulation. The computational cost of a FORM solution, however, is still a few times that of a single, deterministic (numerical) solution. The cost of a reliability analysis using FORM is directly proportional to the size of the random variable space, and to the number of limit state functions. Hence, it is important to consider as random variables only those which present significant contribution to failure probabilities. Sensitivity coefficients can be used to identify these variables.

2.2 Slection of the sampling points

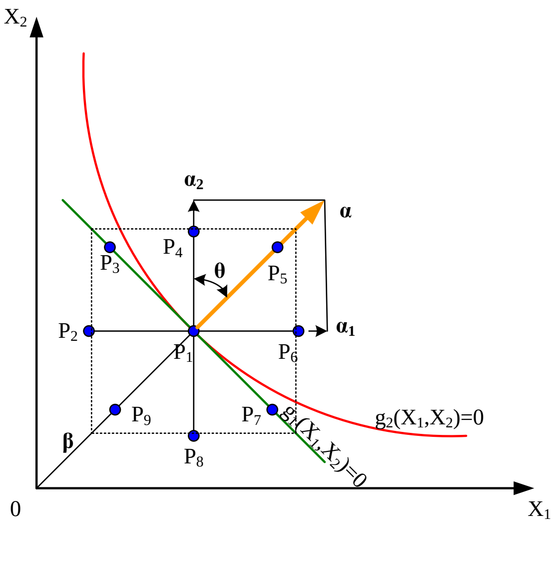

Figure 1: Propossed response surface

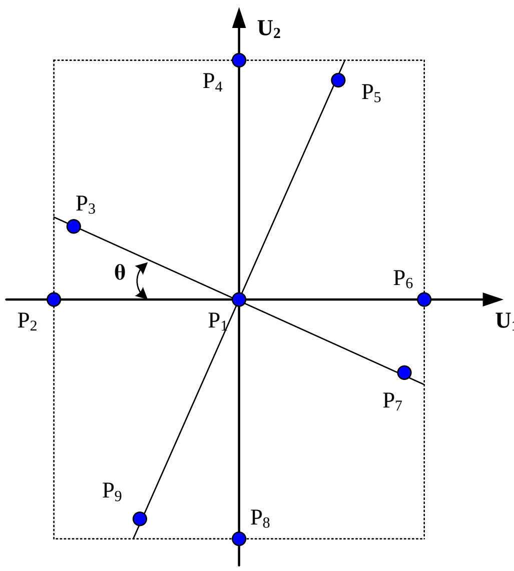

Figure 2: Experimental plan is illustrated for two random variables

The coordinate of the design point are $P_1(\mu_1,\mu_2)$ and the reliability index β is equal to $\sqrt{\mu_1^2 + \mu_2^2}$ (Fig.1). $g_1(X_1,X_2)$ is the fisrt order approximation function and $g_2(X_1,X_2)$ is the second order approximation function.

A quadratic response surface is built considering the design point P_1 as centre of the experimental phan. The coordinate of the sampling points are (Fig 2)

$$\begin{aligned}
&P_1(0,0)\\
&P_2(-f\sigma_1,0)\\
&P_3(-f\sigma_1\cos(\theta), f\sigma_1\sin(\theta))\\
&P_4(0, f\sigma_2)\\
&P_5(f\sigma_2\sin(\theta), f\sigma_2\cos(\theta))\\
&P_6(f\sigma_1,0)\\
&P_7(f\sigma_1\cos(\theta),-f\sigma_1\sin(\theta))\\
&P_8(0,-f\sigma_2)\\
&P_9(-f\sigma_2\sin(\theta),-f\sigma_2\cos(\theta))
\end{aligned}$$

where f>0 is the parameter that identifies the points of the grid and σ_i are standard deviations.

2.3 Compute the sensitivity vector α

The limit state function approximated by the following

$$\tilde{g}_1(\mathbf{U}) = a_0 + \sum_{i=1}^{n} a_i u_i \tag{4}$$

The coefficients a_0, a_i are the solution of the following system of equations

$$A_1\lambda_1 = B_1 \tag{5}$$

Where A_1 is the matrix of the coefficients, λ_1 is the vector of the unknowns a_0, a_i and B_1 is vector of the values of the limit state function in the sampling loints

$$A_1 = \begin{bmatrix} 1 & \mu_1 & \mu_2 \\ 1 & \mu_1 - f\sigma_1 & \mu_2 \\ 1 & \mu_1 + f\sigma_1 & \mu_2 \\ 1 & \mu_1 & \mu_2 - f\sigma_2 \\ 1 & \mu_1 & \mu_2 + f\sigma_2 \end{bmatrix} \tag{6}$$

The vector α is a evaluated on the basis of the gradient vector $\nabla_u \tilde{g}$ of the response surface in the U_space.

$$\alpha = -\frac{\nabla_u \tilde{g}}{\|\nabla_u \tilde{g}\|} \tag{7}$$

The angle θ can be derived from the sensitivity vector α (Fig. 5.2).

$$\begin{aligned} \alpha_1 &= -\sin(\theta) \\ \alpha_2 &= \cos(\theta) \end{aligned} \tag{8}$$

$$\tilde{g}(\mathbf{u}_1, \mathbf{u}_2) = a_0 + a_1 u_1 + a_2 u_2 \tag{9}$$

where a_0,a_1,a_2 are positive constants.

The design point P_1 is located at

$$P_1\left(\frac{a_0 a_1}{a_1^2 + a_2^2}, \frac{a_0 a_2}{a_1^2 + a_2^2}\right) \tag{10}$$

and the vector α has the following components

$$\begin{aligned} \alpha_1 &= \frac{a_1}{\sqrt{a_1^2 + a_2^2}} \\ \alpha_2 &= \frac{a_2}{\sqrt{a_1^2 + a_2^2}} \end{aligned} \tag{11}$$

2.4 RSM-based reliability analysis

The limit state function for a complex structure is an implicit limit state function of the basic random variables X. Their relationship can only be determined through a numerical algorithm, such as the finite element method (FEM). In the response surface method, the actual limit state function g(**X**) is replaced by a polynomial type of function $\tilde{g}(\mathbf{X})$. A quadratic polynomial RS without cross terms is used to approximate the actual limit-state function

$$\tilde{g}_2(\mathbf{U}) = b_0 + \sum_{i=1}^{n} b_i u_i + \sum_{i=1}^{n} b_{ii} u_i^2 \tag{12}$$

where n is the number of random variables X, the unknown coefficients b_0, b_i and b_{ii} are the solution of the following system of equations

$$A_2\lambda_2=B_2 \tag{13}$$

where A_2 is the matrix of the coefficients, λ_2 is the vector of the unknowns b_0, b_i, b_{ii} and B_2 is vector of the values of the limit state function in the sampling loints

$$A_2 = \begin{bmatrix} 1 & \mu_1 & \mu_2 & \mu_1^2 & \mu_2^2 \\ 1 & \mu_1 - f\sigma_1 & \mu_2 & (\mu_1 - f\sigma_1)^2 & \mu_2^2 \\ 1 & \mu_1 + f\sigma_1 & \mu_2 & (\mu_1 + f\sigma_1)^2 & \mu_2^2 \\ 1 & \mu_1 & \mu_2 - f\sigma_2 & \mu_1^2 & (\mu_2 - f\sigma_2)^2 \\ 1 & \mu_1 & \mu_2 + f\sigma_2 & \mu_1^2 & (\mu_2 + f\sigma_2)^2 \\ 1 & \mu_1 - f\sigma_1\cos\theta & \mu_2 + f\sigma_1\sin\theta & (\mu_1 - f\sigma_1\cos\theta)^2 & (\mu_2 + f\sigma_1\sin\theta)^2 \\ 1 & \mu_1 + f\sigma_2\sin\theta & \mu_2 + f\sigma_2\cos\theta & (\mu_1 + f\sigma_2\sin\theta)^2 & (\mu_2 + f\sigma_2\cos\theta)^2 \\ 1 & \mu_1 + f\sigma_1\cos\theta & \mu_2 - f\sigma_1\sin\theta & (\mu_1 + f\sigma_1\cos\theta)^2 & (\mu_2 - f\sigma_1\sin\theta)^2 \\ 1 & \mu_1 - f\sigma_2\sin\theta & \mu_2 - f\sigma_2\cos\theta & (\mu_1 - f\sigma_2\sin\theta)^2 & (\mu_2 - f\sigma_2\cos\theta)^2 \end{bmatrix} \tag{14}$$

Based on the constructed RS, an approximate reliability index β can be computed by solving the following optimization problem

$$\begin{aligned} &\text{Min} && \|\mathbf{U}\| \\ &\text{Subject to} && \tilde{g}(X(U)) = 0 \end{aligned} \tag{15}$$

By applying the HLRF algorithm to solve Eq.12), the approximate reliability index β , probability failure P_f and corresponding design point X_D can be efficiently obtained since the structural responses and corresponding gradients can be calculated. To improve the precision of structural reliability analysis, a sequential framework is usually employed to update the RS through relocating the sampling center using the following linear interpolation. This updating strategy generally can produce a closer point to the actual limit-state function and whereby improve the convergence speed. The process is repeated until the following criterion is satisfied. Accordingly, the numerical procedure of the proposed method is illustrated in Fig.3. Use the design point X_D to find a new centre point X_M for the second interpolation. The new centre point X_M is chosen on a straight line from the mean vector of X to X_D.

$$\mathbf{X_M} = \bar{X} + \frac{(X_D - \bar{X})g(\bar{X})}{g(\bar{X}) - g(X_D)} \tag{16}$$

As shown in Eq. (12), the function $\tilde{g}(\mathbf{X})$ represents the actual function g(**X**) along the coordinate axes X_i. Hence, the points required to obtain $\tilde{g}(\mathbf{X})$ can then be chosen along the axes X_i as the point at the mean vector X of the random variables X_i and 2*n points with coordinates $X_i=\mu_i \pm k_i\sigma_i$ in which k_i is an arbitrary factor and μ_i and σ_i are the mean and standard deviation of X_i and 2*n points is rotated according to the sensitivity vector of the limit state function, respectively (see Fig. 1). Next, the original limit state function is evaluated at the 4*n+1 selected points. A set of linear equations is then formed by using the 4*n+1 values of function $\tilde{g}(\mathbf{X})$ at the selected points.

2.5 Numerical procedure of the proposed method

An iterative procedure is coupled with the response surface in order to achieve a satisfactory approximation of the limit state function (LSF) in the neighbourhood of the design point. This point is determined according to the FORM, evaluating a new response surface and using it at each iteration to predict the new solution.

The iterative procedure is the following:

Step 1. Select random variables **X** and define the state function g(**X**) according to the engineering problem

Step 2. Approximate the limit state function with a first order response surface

Step 3. Compute the gradient $\nabla_U \tilde{g}(u)$ of the response surface

Step 4. Compute the sensitivity vector α

Step 5. Generate 4*n+1 experimental points

Step 6. Approximate the limit state function with a second order response surface

Step 7. Compute the reliability index β

Step 8. Check the convergence

Step 9. Estimate the probability of failure by the importance sampling technique.

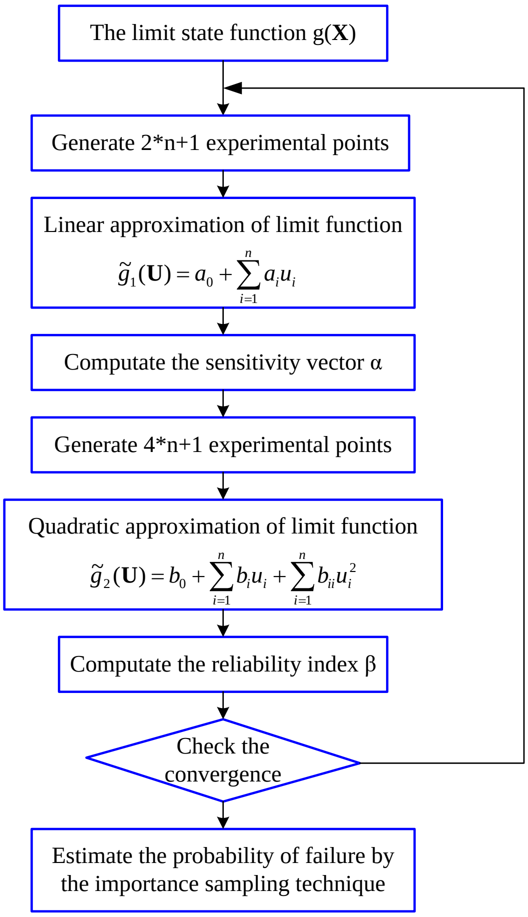

Figure 3. Flowchart of the proposed method

3. Numerical models

3.1 A hypothetical nonlinear limit state

The hypothetical nonlear limit state function is defined as

$$g(\mathbf{X}) = \exp[0.4(X_1 + 2) + 6.2] - \exp[0.3X_2 + 5.0] - 200 \tag{17}$$

where X_1 and X_2 are assumed to be independent and have a standard normal distribution with zero mean and unit standard deviation. The reliability analysis performed by using Directional MCS with 10^6 samples yields 3.623×10^{-3}, and the corresponding reliability index β = 2.685 [12], which are regarded as the exact referenced solution.

The estimates of the reliability estimates strongly depend on the choice of the parameter *f* in the case of the conventional RSM and the proposed method. The results and relative error of the three methods are shown in Table 1 by MCS, the proposed method and traditional RSM. It can be seen that the analysis results of the proposed method are all very close to the exact ones. Propossed method can obtain more accurate P_f results than traditional RSM.

Comparison is made between the proposed method and the conventional sequential RSM in terms of accuracy and efficiency as shown in Table 1. It can be found that the conventional RSM shows a 7.121%, 7.204%, 7.508%, 8.198% deviation for the probability of failure and 0.931%, 0.894%, 0.968%, 1.08% error for the reliability index, whereas the proposed method exhibits only 0.028%, 0.11 %, 0.083%, 0.00% and).00%, 0.037%, 0.037%, 0.00% respectively, which indicates that the present method is more efficient.

Table 1: Results of the reliability analysis of a hypothetical nonlinear limit state

Method	Failure probability P_f	Relative error P_f	Reliability index	Relative error β
MCS	3.623×10^{-3}		2.685	—

RSM (f=1)	3.365×10^{-3}	7.121%	2.709	0.894%
RSM (f=2)	3.362×10^{-3}	7.204%	2.710	0.931%
RSM (f=3)	3.351×10^{-3}	7.508%	2.711	0.968%
RSM (f=4)	3.326×10^{-3}	8.198%	2.714	1.080%
Proposed method (f=1)	3.622×10^{-3}	0.028%	2.685	0.000%
Proposed method (f=2)	3.619×10^{-3}	0.110%	2.686	0.037%
Proposed method (f=3)	3.620×10^{-3}	0.083%	2.686	0.037%
Proposed method (f=4)	3.623×10^{-3}	0.000%	2.685	0.000%

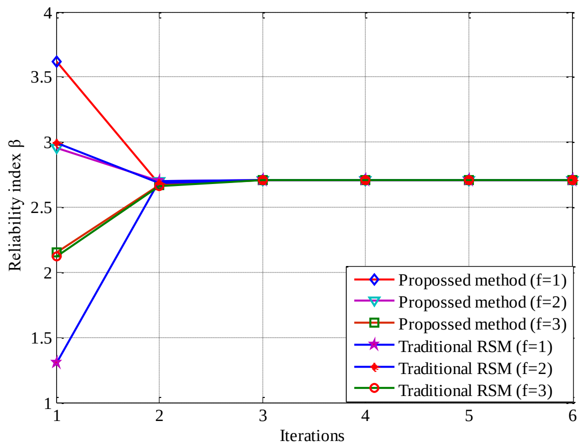

Figure 4: Convergence histories of reliability index for a hypothetical nonlinear limit state

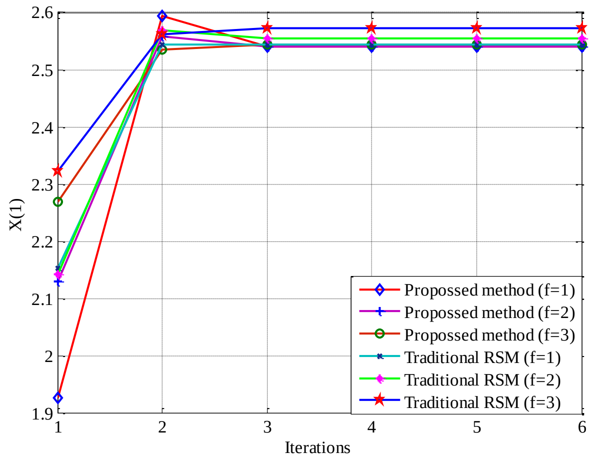

Figure 5: Convergence histories of variable X(1) for a hypothetical nonlinear limit state

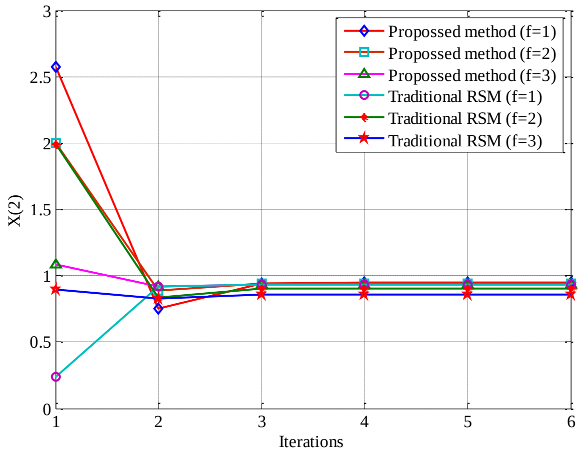

Figure 6: Convergence histories of variable X(2) for a hypothetical nonlinear limit state

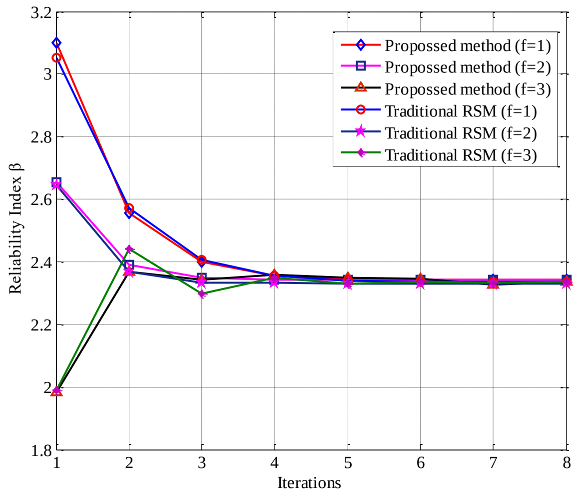

Figure 7: Convergence histories of reliability index for a hypothetical nonlinear limit state

Convergence histories for random variables X_1, X_2 and for reliability index β are presented in Fig. 4, Fig. 5, Fig. 6. Each of the two methods is run another 6 times, respectively, and the estimated reliability indexes are shown in Fig.4. Failure probability results obtained for problem are presented in Table 1. Convergence results presented in Fig. 4, Fig. 5, Fig. 6 compare the histories of the reliability index per iteration of each method. The histories are ploted until satisfaction of the convergence iteration. It can be seen that the random variables X_1, X_2 and reliability index β provided by the proposed method converges faster to the exact value than the traditional RSM. The difference in the first reliability index is due to the number of initial experimental points selected in each of these methods. Traditional RSM starts with 2n+1 experimental points while the proposed method starts with 4n+1 experimental points. Table 1 and Fig. 4, Fig. 5, Fig. 6 compares the reliability index and failure probability resulting from both the traditional RSM and the proposed method with the different parameters *f*. The reliability index and failure probability are also presented to examine the accuracy of the structural reliability analysis results.

3.2 Cantilever beam

This section concerns the deflection limit state of a cantilever beam with rectangular cross section, subject to a uniformly distributed load [13]. The limit state function is written in terms of the maximum vertical deflection of the free end of the beam and a limit value equal to *l*/325, is defined by

$$g = -\frac{wbl^4}{8EI} + \frac{l}{325} \tag{18 }$$

where *w, b, l, E* and *I* are respectively *w* is the load per unit area, *b* is the width of the cross section, *l* is the length of the beam, *E* is the elastic modulus and *I* is the moment of inertia of the cross section. The random variables are the load *w* per unit area and the depth of the cross section are represented respectively by the random variables X_1~N(1000,200) and X_2 ~N(250,37.5). The modulus of elasticity *E* and the length *l* are assumed, respectively, equal to 2 ×10^4MPa and 6m, the limit state function reduces to

$$g(\mathbf{X}) = 18.46154 - 74769.23\frac{X_1}{X_2^3} \tag{19}$$

where X_1 is the load in MPa and X_2 is the depth im mm. The load is assumed to be normal distributed, with a mean μ_1=1000N/m^2 and a standard deviation D_1=200. The depth is also normally distributed, with μ_2=250mm and D_2=37.5.

The exact solution using importance sampling with 10^6 simulations is 9.533×10^{-3}. The estimates of the reliability estimates strongly depend on the choice of the parameter *f* in the case of the conventional RSM and the proposed method. The results and relative error of the three methods are shown in Table 2 by MCS, the proposed method and traditional RSM. It can be seen that the analysis results of the proposed method are all very close to the exact ones. Propossed method can obtain more accurate Pf results than traditional RSM.

Comparison is made between the proposed method and the conventional sequential RSM in terms of accuracy and efficiency as shown in Table 2. It can be found that the conventional RSM shows a 2.349%, 3.546%, 2.602%, 2.895% deviation for the probability of failure and 0.341%, 0.555%, 0.384%, 0.469% error for the reliability index, whereas the proposed method exhibits only 0.357%, 0.304%, 0.629%, 0.053% and 0.085%, 0.043%, 0.128%, 0.000% respectively, which indicates that the present method is more efficient.

Table 2: Results of the reliability analysis of cantilever beam

Method	Failure probability P_f	Relative error P_f	Reliability index	Relative error β
MCS	9.533×10^{-3}	—	2.344	—
RSM (f=1)	9.757×10^{-3}	2.349%	2.336	0.341%
RSM (f=2)	9.871×10^{-3}	3.546%	2.331	0.555%
RSM (f=3)	9.781×10^{-3}	2.602%	2.335	0.384%
RSM (f=4)	9.257×10^{-3}	2.895%	2.355	0.469%
Proposed method (f=1)	9.499×10^{-3}	0.357%	2.346	0.085%
Proposed method (f=2)	9.504×10^{-3}	0.304%	2.345	0.043%
Proposed method (f=3)	9.473×10^{-3}	0.629%	2.347	0.128%
Proposed method (f=4)	9.538×10^{-3}	0.053%	2.344	0.000%

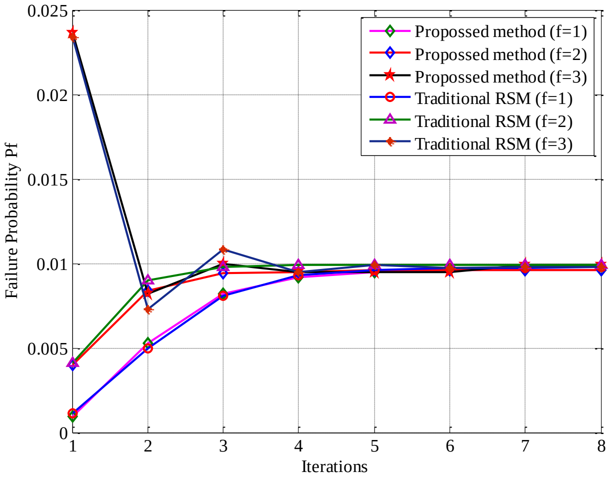

Figure 8: Convergence histories of failure probability for cantilever beam

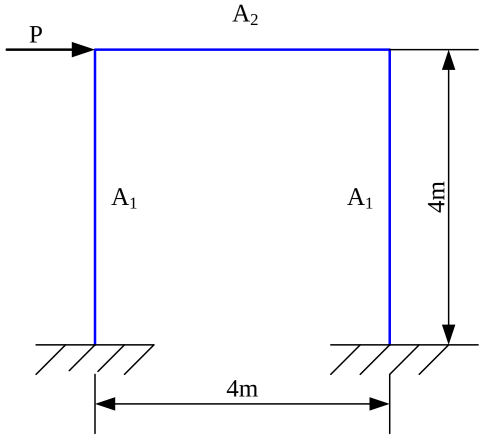

Figure 9: Linear portal frame

Convergence histories for failure probability P_f and for reliability index β are presented in Fig. 7, Fig. 8. Each of the two methods is run another 8 times, respectively, and the estimated reliability indexes are shown in Fig.6. Failure probability results obtained for problem are presented in Table 2. Convergence results presented in Fig. 7, Fig. 8 compare the histories of the reliability index per iteration of each method. The histories are ploted until satisfaction of the convergence iteration. It can be seen that the failure probability P_f and reliability index β provided by the proposed method converges faster to the exact value than the traditional RSM. The difference in the first reliability index is due to the number of initial experimental points selected in each of these methods. Traditional RSM starts with 2n+1 experimental points while the proposed method starts with 4n+1 experimental points. Table 2 and Fig. 7, Fig. 8 compares the reliability index and failure probability resulting from both the traditional RSM and the proposed method with the different parameters *f*. The reliability index and failure probability are also presented to examine the accuracy of the structural reliability analysis results.

3.3 Linear portal frame

This engineering application is a linear frame structure one story and one bay as shown in Fig.9. Different cross sectional areas A_i and horizontal load P are treated as independent random variables, their statistics are listed in Table 3. The sectional moments of inertia expressed as $I_i = \alpha_i A_i^2 \;\; (\alpha_1 = 0.0833,\; \alpha_2 = 0.16670)$. The Young's modulus is treated as deterministic $E = 2 \times 10^6 kN / m^2$.

The limit state function is expressed

$$g(\mathbf{X}) = 0.01 - u_x \tag{20}$$

where u_x denotes the max horizontal displacement as the function of basic random variables. The limit state function is implicit, and the structural response has to be calculated by using the FEM. The reliability index and failure probability calculated by the proposed method is compared with the MCS result with the exact solution $P_f = 0.2322 \times 10^{-2}$ and its corresponding reliability index $\beta = 2.834$ [14]. The results and relative error of the three methods are shown in Table 5.4 by MCS, the proposed method and traditional RSM. It can be seen that the analysis results of the proposed method are all very close to the exact ones. Propossed method can obtain more accurate Pf results than traditional RSM.

Comparison is made between the proposed method and the conventional sequential RSM in terms of accuracy and efficiency as shown in Table 1. It can be found that the conventional RSM shows a 7.121%, 7.204%, 7.508%, 8.198% deviation for the probability of failure and 0.931%, 0.894%, 0.968%, 1.08% error for the reliability index, whereas the proposed method exhibits only 0.028%, 0.11 %, 0.083%, 0.00% and).00%, 0.037%, 0.037%, 0.00% respectively, which indicates that the present method is more efficient.

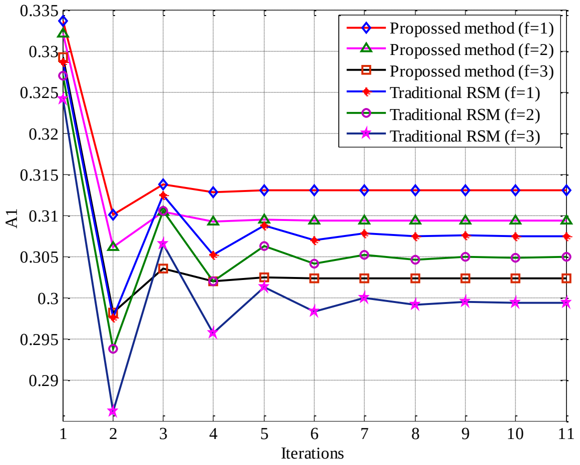

Figure 10: Convergence histories of variable A(1) for linear portal frame

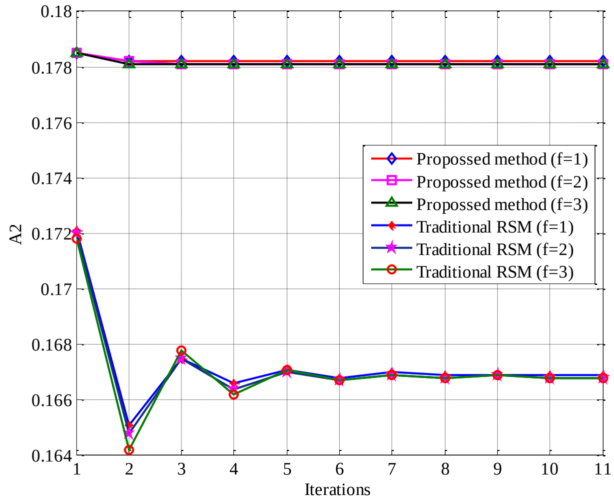

Figure 11: Convergence histories of variable A(2) for linear portal frame

Table 3: Random properties of linear portal frame

Variable	Mean	Standard deviation	Unit	Distribution
A_1	0.360	0.036	m^2	lognormal
A_2	0.180	0.018	m^2	lognormal
P	20.000	5.000	kN	Type I largest

Table 4: Results of the reliability analysis for linear portal frame

Method	Failure probability P_f	Relative error P_f	Reliability index	Relative error β
MCS	2.409×10^{-3}	—	2.819	—
RSM (f=1)	0.614×10^{-3}	74.512%	3.233	14.686%
RSM (f=1.5)	0.610×10^{-3}	74.678%	3.234	14.723%
RSM (f=2)	0.599×10^{-3}	75.135%	3.239	14.899%
RSM (f=2.5)	0.575×10^{-3}	76.131%	3.251	15.325%
Proposed method (f=1)	2.373×10^{-3}	1.494%	2.824	0.177%
Proposed method (f=1.5)	2.360×10^{-3}	2.034%	2.826	0.248%
Proposed method (f=2)	2.361×10^{-3}	1.993%	2.826	0.248%
Proposed method (f=2.5)	2.349×10^{-3}	2.491%	2.827	0.284%

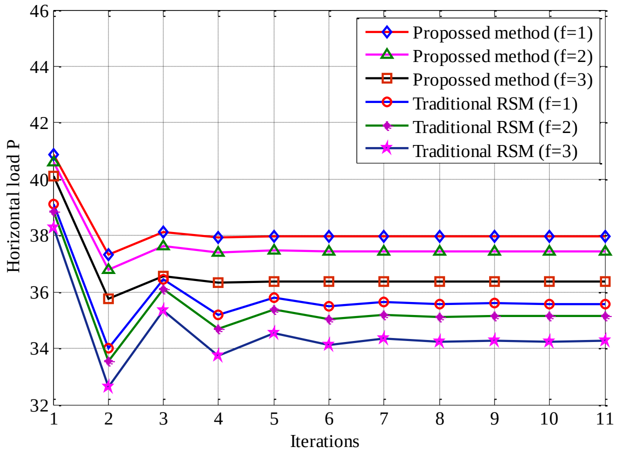

Figure 12: Convergence histories of variable P for linear portal frame

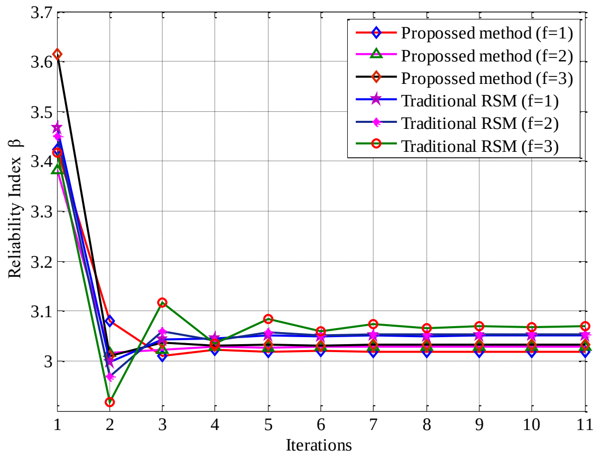

Figure 13: Convergence histories of reliability index for linear portal frame

Convergence histories for random variables A_1, A_2 and P and for reliability index β are presented in Fig. 10, Fig. 11, Fig. 12 and Fig. 13. Each of the two methods is run another 11 times, respectively, and the estimated reliability indexes are shown in Fig.13. Failure probability results obtained for problem are presented in Table 4. Convergence results presented in Fig. 10, Fig. 11, Fig. 12 and Fig. 13 compare the histories of the reliability index per iteration of each method. The histories are ploted until satisfaction of the convergence iteration. It can be seen that the random variables A_1, A_2, P and reliability index β provided by the proposed method converges faster to the exact value than the traditional RSM. The difference in the first reliability index is due to the number of initial experimental points selected in each of these methods. Traditional RSM starts with 2n+1 experimental points while the proposed method starts with 4n+1 experimental points. Table 4 and Fig. 10, Fig. 11, Fig. 12 and Fig. 13 compares the reliability index and MPP resulting from both the traditional RSM and the proposed method with the different parameters *f*. The reliability index and MPP are also presented to examine the accuracy of the structural reliability analysis results.

4. CONCLUSION

An efficient response surface technique is proposed for sensitivity estimation of parameters in structural reliability analysis. At each iteration, the response surface is built after locating 4n+1 sampling points includes 2n+1 sampling points are chosen along the coordinate axes of the U-space of standard normal random variables, as in the classic RSM and 2n sampling points is rotated according to the sensitivity vector of the limit state function. An adaptive procedure in combination with the FORM method is adopted to build successive response surfaces until the convergence. Then, the probability of failure can be computed applying the importance sampling Monte Carlo technique. A better approximation of a limit state function with a reasonable computational effort is the objective of the proposed method. The numerical models showed that the proposed method is able to reach a better approximation in the evaluation of probability of failure than the traditional RSM.

REFERENCES

[1] Zhao W, Liu J K, Ye J J. A new method for parameter sensitivity estimation in structural reliability analysis. Applied Mathematics and Computation, 217, 5298–5306, 2011.
[2] Wang Z L, Huang H Z, Li Y F, Pang Y, Xiao N C. An approach to system reliability analysis with fuzzy random variables. Mechanism and Machine Theory, 52 :35–46, 2012.
[3] Hu C, Youn B D. An asymmetric dimension-adaptive tensor-product method for reliability analysis. Structural Safety, 33: 218–231, 2011.
[4] Guan X F, He J J, Jha R, Liu Y M**.** An efficient analytical Bayesian method for reliability and system response updating based on Laplace and inverse first-order reliability computations. Reliability Engineering and System Safety, 97:1–13, 2012.
[5] Allaix D L, Carbone V I. An improvement of the response surface method. Structural Safety, 33 :165–

172, 2011.
[6] Liu Y. Application of Stochastic Response Surface Method in the Structural Reliability. Procedia Engineering, 28:661-664, 2011.
[7] Okasha N M, Frangopol D M, Orcesi A D. Automated finite element updating using strain data for the lifetime reliability assessment of bridges. Reliability Engineering and System Safety, 99 :139–150, 2012.
[8] Kingston G B, Rajabalinejad M, Gouldby B, Gelder P H A J M V. Computational intelligence methods for the efficient reliability analysis of complex flood defence structures. Structural Safety, 33:64–73, 2011.
[9] Jiang C, Han X, Lu G Y, Liu J, Zhang Z, Bai Y C. Correlation analysis of non-probabilistic convex model and corresponding structural reliability technique. Comput. Methods Appl. Mech. Engrg, 200 :2528–2546, 2011.
[10] Weitao Zhao, Zhiping Qiu. An efficient response surface method and its application to structural reliability and reliability-basedoptimization, Finite Elements in Analysisand Design, 67:34-42, 2013.
[11] Yanjun Ou, Yeting Wu, Jun Cheng, Yangyang Chen, Wei Zhao. Response Surface Method for Reliability Analysis Based on Iteratively-Reweighted-Least-Square Extreme Learning Machines, Electronics, 7:2-15, 2023.
[12] Hurtado J E. Dimensionality reduction and visualization of structural reliability problems using polar features. Probabilistic Engineering Mechanics, 29:16–31, 2012.
[13] Hurtado J E. Dimensionality reduction and visualization of structural reliability problems using polar features. Probabilistic Engineering Mechanics, 29:16–31, 2012.
[14] Gupt S, Manohar CS. An improved response surface method for the determination of failure probability and imfortance measures. Structural safety, 26:123-139, 2004.

MỘT KỸ THUẬT BỀ MẶT PHẢN ỨNG HIỆU QUẢ ĐỂ TÍNH ĐỘ NHẠY TRONG PHÂN TÍCH ĐỘ TIN CẬY KẾT CẤU

CHÂU MINH QUANG
Viện Đào tạo quốc tế và Sau Đại học – Trường Đại học Công nghiệp Thành phố Hồ Chí Minh
**Tác giả liên hệ: chauminhquang@iuh.edu.vn*

Tóm tắt. Phương pháp RSM là một phương pháp có độ tin cậy về kết cấu mạnh mẽ sử dụng các giá trị của hàm tại các điểm cụ thể gần đúng với hàm trạng thái giới hạn bằng biểu thức đa thức. Hàm phân tích thay thế hàm trạng thái giới hạn chính xác mà thời gian tính toán cần thiết để đánh giá độ tin cậy của hệ kết cấu có thể giảm đáng kể. Tuy nhiên, vị trí của các điểm mẫu đã được một số tác giả nghiên cứu và hiệu quả của phương pháp bề mặt đáp ứng vẫn đang được thảo luận. Vì vậy, nghiên cứu này đề xuất một phương pháp bề mặt đáp ứng mới để tính độ nhạy của các tham số trong phân tích độ tin cậy kết cấu. Đa thức thứ nhất không có số hạng chéo được sử dụng để tính gần đúng hàm trạng thái giới hạn và có thể thu được vectơ độ nhạy của hàm trạng thái giới hạn. Thiết kế thực nghiệm với 4n+1 điểm lấy mẫu bao gồm 2n+1 điểm lấy mẫu được chọn dọc theo các trục tọa độ trong không gian U của các biến ngẫu nhiên chuẩn, như trong mô hình RSM cổ điển và 2n điểm lấy mẫu được quay theo vectơ độ nhạy của hàm trạng thái giới hạn được xây dựng. Một đa thức bậc hai được sử dụng để tính gần đúng hàm trạng thái giới hạn và có thể thu được điểm có xác suất lớn nhất (MPP) bằng cách tiến hành thuật toán HL-RF dựa trên RS đã tạo. Để cải thiện hơn nữa độ chính xác của phân tích độ tin cậy, Mô phỏng Monte Carlo (MCS) được tiến hành trên đa thức đã thiết lập để tính xác suất thất bại. Các mô hình số được xét để chứng minh những ưu điểm của phương pháp được đề xuất.

Keywords: phân tích độ tin cậy, phương pháp bề mặt đáp ứng, phương pháp độ tin cậy bậc nhất, phương pháp phần tử hữu hạn, hàm phản hồi ngầm.

Received on: 25/4/2024

Revised on: 10/7/2024